Physical Parameters of Substorms from Automated Forward Modeling

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Automated Forward Modeling (AFM) is an inversion technique based on magnetic data alone, that determines physical parameters associated with electric current flow in near-Earth space. In its simplest form, it may be used to invert magnetic perturbations observed along a meridian. The total electric current crossing the meridian may be determined, as well as the latitudes between which it flowed. The technique is based on nonlinear optimization of the parameters of a forward model. It is possible to compare model output to the original input to ensure that the routine has functioned well and that output parameters are reliable and correspond to physical reality. Characteristic behaviors of substorms are readily seen in modeling output: the current strengthens rapidly and considerably at an expansive phase onset, with motion of the electrojet borders. In many cases the poleward border moves poleward rapidly. The recovery phase is accompanied by a decrease in the current across the meridian, but we do not find indications of poleward retreat of the auroral oval on up to a two-hour timescale. AFM output parameters show the timescale associated with current rise to be approximately 30 minutes while that associated with the rapid expansion of the poleward border of the substorm current system is 5 minutes. In addition, we find that average characteristics of the current deduced in our models closely follows that of the AL index, while in individual events the two often do not match. The motion of borders of the electrojet is very similar to that deduced for the electron aurora from satellite studies. AFM allows both the current strength and the borders to be determined from ground magnetic data alone, which is generally available on a continuous basis.

1. Introduction

The expansive phase of substorms remains unexplained and in some ways may yet be poorly characterized. Ground magnetic perturbations offer a way to study the currents associated with substorms, but their interpretation in a quantitative fashion has until recently been poorly developed. A widely used measure of substorm

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strength has been the AL index, a composite of the negative X perturbations from globally spaced auroral zone stations. This index is in nT and does not represent a clear physical parameter at any given point. Recent work to improve the situation includes, for example, inversion near magnetometer arrays by Pulkkinen and coworkers [2003] to determine equivalent ionospheric currents. It is also possible to use auroral imaging, notably from space, to obtain some geometric parameters associated with the auroral oval during substorms [Mende et al., 2003]. A long-standing paradigm for the three-dimensional structure of currents in near-Earth space associated with substorms is the substorm current wedge (SCW) [McPherron, 1972]. In this system, field-aligned current flows into the morning sector and out of the evening sector. These regions are joined by a region of electrojet flow in the ionosphere crossing the midnight meridian. The electrodynamics of the substorm-activated region can be studied with averaged models [Gjerlov and Hoffman, 2002] to include Pedersen currents which flow on average north-south, but much of the magnetic field observed in the auroral zone arises from Hall currents roughly corresponding to those in the SCW paradigm. In this study we use what amounts to a current wedge but focus on local ionospheric currents near the middle of the SCW. From these we may determine the latitudinal extent of the electrojet as well as its strength, for individual substorms. These simple physical parameters may be compared to values obtained previously, the strength of the electrojet being compared to AL, and the borders to those determined from imaging studies. Familiar behaviors are quantified and agree on average with those already known. In individual substorms, the parameters deviated significantly from the averages. These parameters are derivable from readily and continuously obtainable magnetic data and should be useful in future studies. For example, we do not directly consider interplanetary parameters in this work, but the parameters we derive could easily be used to facilitate studies of their relation to substorms.

2. Automated Forward Modeling (AFM)

Interpretation of ground magnetic data is difficult, even if the data come from the same magnetic meridian. In Fig. 1 the X (northward) and Z (downward) components of the magnetic field are shown for three auroral zone stations on the Canadian Churchill meridian for an event on Feb. 8 1997. As was true for all data processed in this study, baselining was done by selecting periods of quiet days throughout the year and in the bracketing December and January. A Chebyshev fit was done for each minute of the day as a function of day number. A quiet day curve for any day could be calculated, in which each minute was based on different quiet days. This quiet day curve was smoothed with a time constant of about two hours and the result subtracted from data for any given day. By examining variation in the synthetic quiet days, we estimated that our quiet days were accurate only to about 12 nT, despite the investment of effort in making them. For the present study of active periods, this degree of accuracy is more than sufficient. In Fig. 1, data for Ottawa is also shown for comparison. This data was not used in modeling and was baselined simply with respect to a relatively quiet period at 0 UT. The auroral zone stations show variations in X and Z characteristic of substorms. The X component signatures largely arise from a westward electrojet across the meridian. Activity of this current system began about 10 UT and until 13 UT the X perturbations were strongest over Fort Churchill. This localiztion is supported by the near-zero Z component at that station, flanked by negative Z south of it and positive Z north of it. At 13 UT an increase in current strength took place as inferred from the increasingly negative X signature. The strengthening Z component at Fort Churchill and Eskimo Point after onset gives some indication of both increasing current and of poleward motion of the electrojet, and after 13:30 UT, the Z decreases indicate that the center of the electrojet had passed overhead. At subauroral station Ottawa, the Y (eastward) component signature is that of an overhead downward FAC. Changes in the X component there likely indicate changes in the position of the overall current system; the Y component is at a maximum under the FAC and does not change rapidly with changes in position of the current. We will return to this point below. Suffice it to say that examination of the ground magnetic data allows some inferences to be drawn about the current systems, but that these are qualitative and obtained through subtle reasoning.

Within the standard SCW paradigm, one might expect that a simple current system could represent much of the substorm onset phenomenology. Automated Forward Modeling proposes a simple forward model of current systems which could give rise to the magnetic perturbations observed. The parameters in that model are varied in such a way that the deviation between the observed magnetic fields and those predicted by the model are reduced. In the ideal case, the parameters can be chosen to correspond to simple physical parameters associated with the current system. The forward model is made using the Biot-Savart law in combination with Earth induction, and specifying where currents flow in space and the ionosphere [Kisabeth and Rostoker, 1977; Kisabeth 1979]. Adjustment of the parameters specifying the current system is done until the match to the input data is optimal. In principle, arbitrarily complex current systems may described in three dimensions in near-Earth space and their parameters determined. In practice, available magnetic data is sparse and well-determined solutions can be difficult to obtain.

The optimum situation can be found when, as in this work, data from meridian chains is available, since in many cases a simple model involving an electrojet flowing across the meridian chain is physically realistic, and then there is a good ratio of data available to parameters to be determined. From meridian chain data a forward modeling procedure can give the current across the meridian and the latitudes between which it flowed. In this way the many data points specifying the magnetic perturbations along a meridian can be reduced to three simple parameters, which have an easily understood physical significance. For efficient processing of large amounts of magnetic data, the matching process can be automated. In the Automated Forward Modeling (AFM) procedure this is done using the Levenberg-Marquardt algorithm [Press et al., 1992]. A schematic of the variables involved and the way in which they are varied is shown in Fig. 2. Detailed descriptions of the AFM procedure are given elsewhere [Connors, 1998; Connors and Rostoker, 2002].

When the AFM technique was applied (automatically, with no initial parameters supplied) to the data of Feb. 8 1997, the results in Fig. 3 were obtained. The top panel shows the borders of the electrojet. The bottom panel shows the eastward current, negative since this is in fact a westward electrojet. It is apparent that at about 10 UT, a short-lived intensification took place which affected the electrojet borders and caused a strengthening of the current across the meridian. After this a classical growth phase took place, with equatorward motion of both electrojet borders. In the late stage of the growth phase, some strengthing of current took place. At 13:05 UT, rapid motion of both electrojet borders took place, most notably poleward motion of the northern border, followed very shortly by an abrupt strengthening of current. This is a well-recognized signature of auroral breakup, i.e. expansive phase onset. Both electrojet borders moved poleward and the current strengthened until about 14 UT. After this the current decreased and both electrojet borders remained at high latitude on a two-hour timescale. Also plotted in Fig. 3 is the provisional AL index. Notably between 12 and 13 UT, it does not match very well the trend of current across the Churchill meridian. Inspection of data from Alaska station College shows a very close resemblence to the AL index, with X values smaller during this period than at Fort Churchill and thus able to dominate the index, which is a lower envelope. Thus during the pre-onset period the AL index does not match the meridian current since the index in fact reflects activity several time zones away. To support the validity of the meridian current results, we note the strong resemblence of the current determined from AFM to the Y value at Ottawa seen in Fig. 1. This relationship was made more quantitative by using AFM with fixed input parameters similar to those used for meridian modeling, with the only free parameter being the current. In this case, if the electrojet (and FAC sheet intersection with ionosphere) latitudes were fixed between 62.6 and 66.8 degrees, a current weaker than that obtained in meridian modelling was obtained. This corresponds to having the FAC too close to Earth. If these latitudes were fixed at 66 to 70 degrees, a current almost identical to that obtained from the meridian modeling was obtained. Thus for reasonable latitudes of the electrojet/FAC intersections, the same current was obtained from independent modelling based on the meridian and based on the midlatitude Y at Ottawa. This supports the concept that the net (see Gjerlov and Hoffman [2002]) FAC and the cross-meridian current are the same, as they would be in the SCW paradigm.

Having established that the current parameter has a connection to AL that may not be exact in individual cases, we proceed to examine the statistical relation of AFM output parameters and known results.

3. Statistical Properties of Substorm Currents and Boundaries

AFM was applied on the CANOPUS Churchill meridian (336° magnetic) to invert the whole year of 1997. This meridian chain includes those stations shown with centred dipole latitudes given in Fig. 1, and in addition to the south the stations Island Lake at 63.7°, and Pinawa at 60.0°, and to the north Rankin Inlet at 72.8° and Talotayak at 80.4°. Contributions from each station were weighted equally but the usually small perturbations at the northernmost and southernmost stations are generally not determinant in the inversions. The inversion routine nominally uses all magnetic components but wieghtings are applied, and with those weightings the effects of only X and Z perturbations were used in the meridian line modeling. Output plots similar to Fig. 1 but covering 24 hours were generated for each day of the year. These output plots were manually examined to select substorms which met the following criteria: good convergence had to have been obtained as indicated by little scatter in the output points; a good match had to be evident to the input data when the results from the inverted parameters were overplotted with the data; and there had to be abrupt and near-simultaneous changes in the electrojet borders coupled with increase in current. The event of Feb. 8 1997 shown in Fig. 1 was anomalous in the sense that the growth phase currents were strong and allowed convergence of the routine before onset. In general the growth phase currents were weak so that the modeling results were of acceptable quality only at and after onset. Thus we do not discuss quantitative aspects of growth phase in this paper.

3.1. Location of Substorm Current Wedge

Using the criteria outlined above, the times of onset were determined for 75 separate onsets on 63 days. These correspond to times at which the ionospheric currents of the SCW were overhead from the Churchill meridian. Frey et al. [2004] used Image FUV images to determine the "substorm onset location", by which they in fact mean "auroral breakup location". They found the mean MLT of onset to be in the late evening sector at 23.0 MLT. To compare to our times of onset, we note that the Churchill meridian is seven times zones west of Greenwich. Thus the magnetic local time is seven hours behind UT, or MLT = UT - 7. Thus the UT at the Churchill line when an onset takes place is MLT + 7, and Fig. 4 shows the Frey et al. [2004] distribution, taken from their released data table and corrected modulo 24, with a clear peak at 6 UT. The results of this study are also shown on the same time scale, and clearly have a centroid very close to 7 UT, i.e. local magnetic midnight. Frey et al. [2004] compared their results to those of several other imaging studies, and although some seasonal effects were seen, they were consistent with those studies. The difference here can be understood if one bears in mind that the image studies look at auroral brightenings and that these correspond in large part to the emissions due to precipitating electrons associated with upward field-aligned currents in the evening sector. The magnetic studies, on the other hand, are not biased toward one end of the SCW but toward its middle. Indeed the distribution has a long tail toward later UTs (the east) as it is well known that the westward electrojet, whose changes are detected, can extend (in fact originate from) far into the morning sector.

The latitude of an onset was determined as the average position of the electrojet borders at the time of onset. An advantage of using magnetic data is that other parameters of interest at the time of onset may be determined. In this case we have determined the current at time of onset, readily determined from the modeling result files represented by Fig. 3. It is well known that under active conditions the auroral oval expands and has larger currents associated with it. Thus we expect that the onsets starting at more southerly latitudes should have larger currents (at time of onset) associated with them, regardless of what the peak current later attained may be. This is borne out in Fig. 5 where the trend to have larger currents for more southerly onsets is clear. With a dataset of this limited size we have not deemed it useful to make a fit to this trend, but the trend is clear and as expected. The distribution of current at time of onset is clearly highly biased to small currents, under 0.1 MA. The distribution of latitude itself may be compared with the UV optical results of Frey et al. [2004]. The histogram at the bottom of Fig. 5 shows a clear peak just above 65 degrees while the Frey et al. results are peaked at 66.4 degrees geomagnetic. This constitutes basic agreement of these results given that the coordinate systems are slightly different and the disparate data types. Clearly a more comprehensive study using magnetic data could be fruitful, providing a basic relation subject to quantitative analysis and comparison to models.

3.2. Relation of Current to AL Index

The AL index has been used in numerous statistical studies of geomagnetic phenomena. The index attempts to give a global view by taking the lower envelope of magnetic perturbations at several stations in the auroral zone. It is in units of nT and nonlocal. The example of Feb. 8 1997 shown above indicates that it did not correspond in that case to the currents across the Churchill meridian. However one wonders whether the average meaning of AL may nevertheless have the average of post=onset currents across a given meridian in the auroral zone. This hypothesis can readily be tested.

Superposed epoch studies have been used to study the relation of solar wind to substorms in a statistical sense where many events are summed so that random variations in parameters tend to cancel out while significant ones reinforce. An early example with the AE index is Foster et al. [1971] and a recent study with the AL index is Hsu and McPherron [2004]. We have done superposed epoch studies in order to relate currents and AL, using the time of onset as the reference time. Fig. 6 shows the average value of current at each minute in the two hours following onset for 63 substorms, plotted as a function of the corresponding Provisional AL value. The averaged data allow a fit to be done with a correlation coefficient of 0.942. The average current is related to average AL by the relation $I(MA) = -0.032162 - 0.001653 * AL$. We note that the AL value would be negative even for 0 current across the meridian. This may be attributed to AL incorporating distant currents, so that it is biased negative with respect to the local conditions.

Superposed epoch studies have also been used to simply characterize the substorm and its time constants, which could then be related to parameters of a proposed model system. We repeated the characterization done by Weimer [1993] using physical parameters rather than an index and average currents and fit are shown in Fig. 7. We found that the current is well represented by the parametrization $I(MA) = c_0 + c_1 t e^{pt}$. We found c_0 to be 0.151 MA, c_1 1.63 MA/h, and p to be -1.98 h. The parameter that can be directly related to Weimer's parameters is p, which he found to have a value of -2.12 for his median class (AL=-400 to -600 nT) of substorm. He also found a peak in AL at 27 minutes after onset, which is very close to the value evident for the peak of current in Fig. 7. We have used our conversion factor for AL to current to overplot the current derived from the superposed epoch AL. This result would also be well fit by the parameters obtained by fitting the currents, i.e. these points strongly resemble the average current observed.

3.3. Optical and Magnetic Auroral Oval Boundaries

Inversion of magnetic meridian data supplies the electrojet boundaries in addition to the strength of current in a meridian. Fig. 8 shows these boundaries for 63 substorms. It is notable that the poleward border on average moves poleward and by the time the current is maximal about one half hour after onset, has essentially reached its maximal latitude. On the other hand, the equatorward boundary does not change on average, this despite cases to the contrary such as that of Feb. 8 1997 presented above. Results for auroral brightness boundary changes from 91 substorms studied by Mende et al. (2003) are also presented. These represent the electron induced auroras as observed by the Image WIC instrument, and in the meridian of onset of brightening. It is seen that the behavior is identical to that of the magnetic electrojet signatures, although the width of the optical zone of activity is three degrees larger. There may be some offset between the coordinate systems used, so that we do not attach significance to the coincidence of the poleward border. A fit was done to the poleward magnetic border movement, that at the equatorward boundary being considered negligible. That border is well parametrized by the curve $67.5 + 4.21 * (1.0 - e^{-5.47*t})$ (in centered dipole coordinates). Obviously this fit, possibly with a base level shift, would parametrize well the (electron) optical results also.

4. Conclusions

We have given a brief description of how a forward model of auroral zone currents may be optimized with respect to ground magnetic data to give simple parameters about electrojets. We have shown a relation to midlatitude currents and to average values of AL that justify the statement that a good approximation to the actual current across a meridian is obtained with this method. Comparison to optical results suggests that the behavior of electrojet boundaries is well modeled also, although there are discrepencies in the width of the magnetic and optically defined electrojets. In turn we find that use of the AL index, although clearly not justified in each event studied, on average does give information about auroral zone currents, and we gave a conversion equation between AL and current. With one method we have reproduced time constants and behaviors deduced from previous magnetic and optical studies. We suggest that AFM is a powerful tool in studying onsets and plan larger and more comprehensive studies using it.

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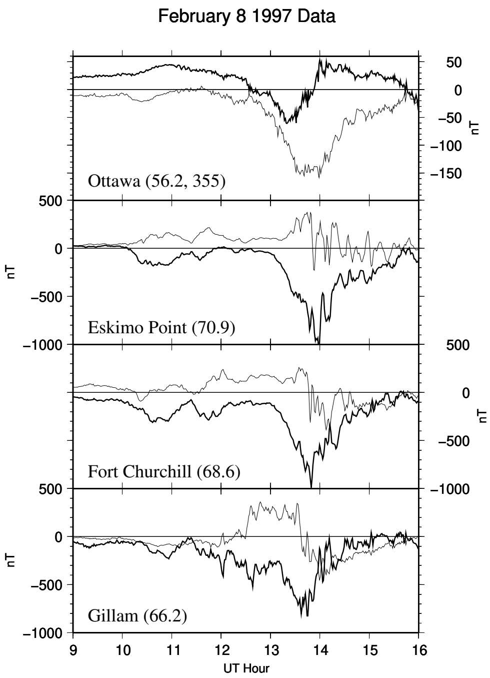

Figure 1. Magnetic data for the growth phase and onset of a substorm on Feb. 8 1997. The bottom three panels show the X (heavy line) and Z perturbations at the successively more northward stations of Gillam, Fort Churchill, and Eskimo Point of the CANOPUS Churchill line. The centered dipole magnetic latitude is indicated after each station name. The top panel shows the X (heavy) and Y perturbations at Ottawa, which lay approximately under the downward FAC during the main disturbance.

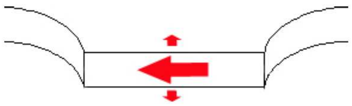

Figure 2. By varying the current (large horizontal arrow) and latitudinal boundaries (small vertical arrows) an optimal match of model results and data can be made and these physical parameters determined. Field-aligned currents may be included (as shown at ends of ionospheric current flow region).

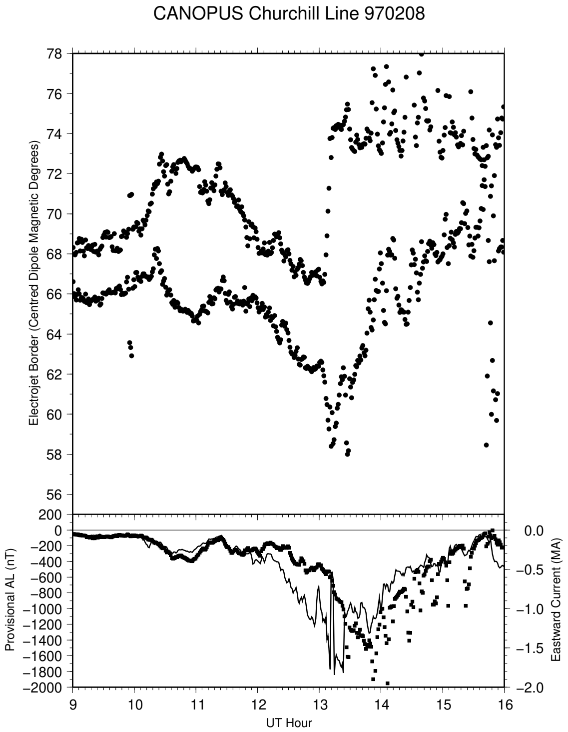

Figure 3. AFM inversion results for a substorm on Feb. 8 1997. Upper panel shows the north and south boundaries of the electrojet as it traversed the Churchill meridian. Bottom panel shows total current across the meridian (dots) and the AL index (trace). The growth phase is clear, with equatorward motion of the electrojet boundaries. The subsequent substorm onset is marked by rapid poleward motion of the north boundary of the electrojet, somewhat slower poleward motion of the south boundary, and an increase in current across the meridian.

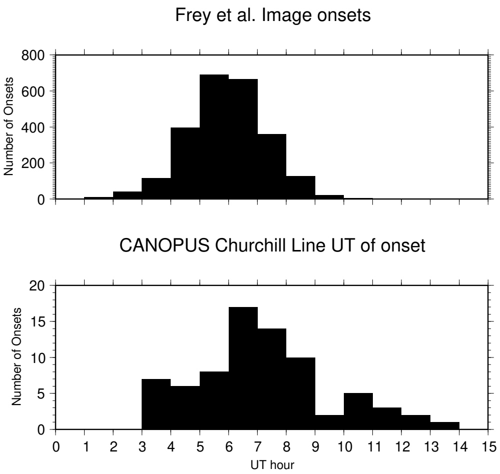

Figure 4. Distribution of onsets in UT. The top panel shows the distribution of Frey et al. [2004] corrected to the meridian of Churchill. The bottom panel shows the times of onset actually observed at Churchill. The difference is attributed to use of optical vs. magnetic data.

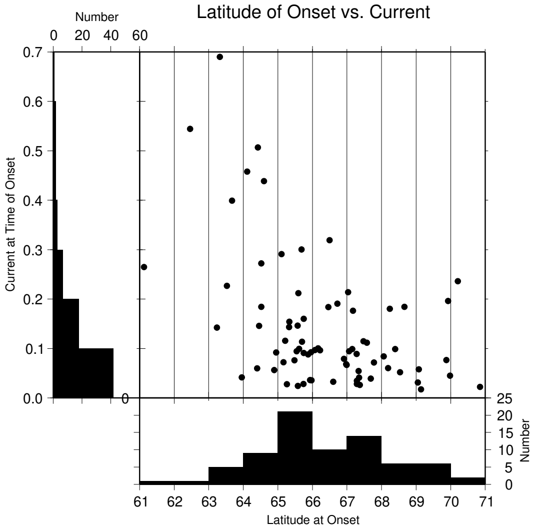

Figure 5. Distribution of onsets in latitude as a function of current at time of onset. The main panel is a scatter plot of current across the meridian against latitude, at the time of onset. The horizontal histogram at the bottom shows the average onset latitude in one degree bins, while the vertical histogram at left shows the average current at the time of onset in 0.1 MA bins.

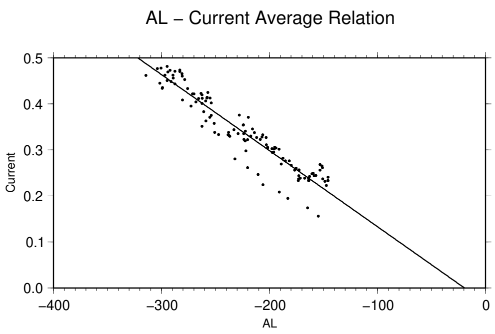

Figure 6. (AL,current) pairs for 120 post-onset minutes averaged over 63 substorms in 1997. The best-fit line is indicated.

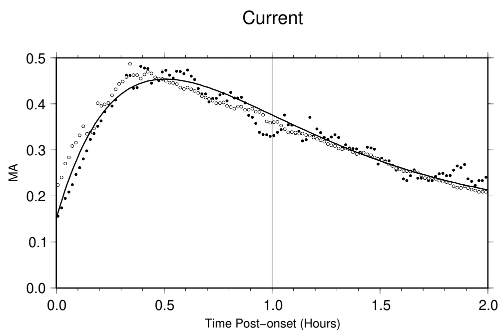

Figure 7. Averaged profile of current across the meridan for 75 onsets. Solid dots show the current each minute after onset. The line is the fit to the current data by a curve based on the Weimer [1993] parametrization of AL. Open dots show the corresponding AL indices scaled to current using the relation derived from Fig. 6.

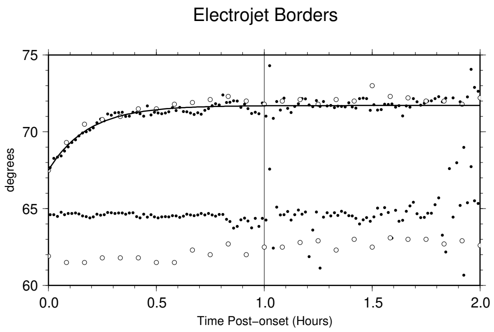

Figure 8. Small filled dots indicate the electrojet borders as derived from averaged results for 63 substorms. Larger open dots represent results from 91 optical (electron derived WIC) onsets.