A Centenary Survey of Orbits of Co-orbitals of Jupiter

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Abstract: Jupiter's Trojan asteroids fulfill the prediction of Lagrange that orbits can be stable when a small body orbits in specific locations relative to its 'parent' planet and the Sun. The first such Trojan asteroid was discovered slightly over one hundred years ago, in 1906, and subsequently similar asteroids have been discovered associated with Mars and with Neptune. To date no Trojans have been discovered associated with Earth, but several horseshoe asteroids, co-orbital asteroids moving along a large range of the Earth's orbit, have been found. Other planets also are not known to have Trojan-type asteroids associated with them. Since the number of detected Jupiter Trojans has increased dramatically in the last few years, we have conducted a numerical survey of their orbital motions to see whether any in fact move in horseshoe orbits. We find that none do, but we use the enlarged database of information about Trojans to summarize their properties as now known, and compare these to results of theory.

1. INTRODUCTION

Since the first Jupiter Trojan was found in 1906, many other similar objects, following or preceding Jupiter along its orbit, have been discovered. The three possible classes of co-orbital motion, Trojan libration near a Lagrange point, horseshoe motion along the planet's orbit, and quasisatellite libration in the vicinity of the planet, have now been observed in the Solar System. However, most objects associated with Jupiter appear to be Trojans. The number of co-orbital companions of Jupiter being very large, that they should all be restricted to this class of motion deserves investigation. The increase in number of known Jupiter Trojans has been dramatic in the past decade, as observing and computing power available to astronomers has improved. Despite large amounts of theoretical work for Jupiter Trojans, the long-term dynamics of individual real objects, which can be related to questions of origin and fate, has not been investigated in detail. To study this requires a large-scale numerical survey of the Jupiter Trojan swarms, and an attempt to classify and group the motions currently undergone by these bodies. We have done this with special attention to asteroids potentially undergoing horseshoe motion in Jupiter's co-rotating frame.

2. BACKGROUND

The first of Jupiter's Trojan asteroids, 588 Achilles, was discovered by Max Wolf at Heidelberg on February 22, 1906 (Wolf, 1906) using photographic techniques. Modern ephemerides give its magnitude on that date as about 15.3. Wolf noted its slow rate of motion (which would be as compared to Main Belt asteroids) at once but does not appear to have made special efforts to follow the object. He originally called the object 1906 TG and by the time of publication of its naming as Achilles (Wolf, 1907), the two other Trojans Patroclus (1906 VY) and Hector (1907 XM) had been found and recognized as a group of "sonnenfernen" (distant from the Sun) objects. These Trojan asteroids, by being at very nearly the same semimajor axis as Jupiter, approximately share its mean motion (1:1 resonance). Since interactions with other bodies are minimal, Trojan motion is usually explained analytically in terms of three-body theory. This was pioneered by Lagrange, who in 1772 presented his "Essai sur le problème des trois corps" to the Paris Academy in a prize competition. One aspect of such motion was the presence of points of stability now referred to as Lagrange points. Lagrange apparently believed his mathematically elegant theory, some aspects of which are discussed below, to be of theoretical interest only (Wilson, 1995). Shortly after Wolf's (1906) announcement, Charlier (1906) at Lund made the connection to Lagrange's theory, including suggesting a libration period of 148 years. The finding of 1906 VY (Patroclus) allowed Charlier (1907) to make this connection yet firmer, identifying this object to lie at one Lagrange point (L_4) while Achilles lies at the other (L_5). Subsequent discoveries in the intervening century have allowed subtle aspects deriving from Lagrange's theory to be examined. Trojan motion can also be viewed in terms of resonance. Resonances can serve to destabilize certain orbits, such as the Kirkwood gaps found in the main belt. Conversely, resonances are also capable of producing stable orbits, as is the case with the Trojans, in

1:1 resonance with Jupiter. Already shortly after the discovery of the Trojans, the tentative explanation of the Kirkwood gaps in terms of resonance, and a similar approach to Trojan motion, was established (Brown, 1911).

2.1 Three Body Models

The three body model is an approximate one which contains only the Sun, a planet, and an asteroid which behaves as a test particle. In the 'restricted' three body problem, these bodies are considered to move in a plane and the Sun and planet to move in circular orbits. According to Lagrange's work of 1772, the restricted three-body model predicts five positions relative to the planet where the test particle can remain in a stable orbit (Murray and Dermott, 1999). These are called Lagrange points, and traditionally denoted by the capital letter L with a subscript. L_1 lies between the planet and the sun, L_2 lies behind the planet on a line connecting it to the Sun, and L_3 lies directly opposite the planet on the other side of the Sun. The contours of effective potential near L_1, L_2 and L_3 are saddle shaped, while those near L_4 and L_5 are bowl shaped wells. Only L_4 and L_5 are stable to small perturbations. These points lie 60º away from the planet along the orbit.. Zero-velocity curves arising in the restricted three-body problem (Fig. 1) are related to the effective potential but do not represent the actual orbits of small bodies. Nonetheless, they outline two classes of motion, which are tadpole orbits and horseshoe orbits, based on the appearance of the associated zero-velocity curves in this diagram. A third class of co-orbital motion is now known, the quasi-satellite, in which the small body moves in the gap near the planet (Mikkola et al., 2006). In the case of Jupiter, only Trojan asteroids have been found to date. In the case of Earth, likely due to observational selection effects, only horseshoe objects and quasi-satellites are known (Brasser et al. 2004). In the case of Mars, it has recently been realized that both Trojan and horseshoe objects exist (Connors et al., 2005). One of our aims here was to more closely examine the large number of presumedly Trojan orbits associated with Jupiter to see if any were in fact horseshoes. The suggestion that such orbits could exist had been made already by 1913 (Einarsson, 1913) when only four Trojans were known and evidence of libration was first measured.

Recent work on the possibility that Jupiter and Saturn themselves were once in a 2:1 mean motion resonance (Morbidelli et al., 2005) suggests an extremely dynamic history for the Trojans. The current near-resonant mean motion ratio is about 2.5:1 (the "Great Inequality"). The 2:1 resonant situation would have arisen early in the history of the Solar System; before it there would have already been Trojans left from the formation of the system and these would have been completely dispersed due to it. The present Trojans would have been captured from distant regions after the resonant condition ended. This theory explains the large inclination distribution of the present Trojan clouds, and why Trojans should have comet-like compositions as is usually observed, although the latter property is not highly diagnostic of source region (Barucci et al., 2002).

The known Jupiter Trojans lie in two main swarms along Jupiter's orbital path (Fig. 2). Associated with their large inclinations of up to and beyond about 30º, they have a noticeable vertical dispersion. Despite this tendency to large inclinations, the theory of three body motion still can be applied, with some complications beyond those of the restricted problem but with similar results, as will be discussed below. Characteristic Trojan orbits as seen in Jupiter's co-rotating frame (Fig. 3) may be thought of as having an epicyclic motion with a period similar to that of Jupiter, superposed on a longer term libration (Murray and Dermott, 1999). The latter results in systematic variations in the osculating parameters, including the semimajor axis. These variations are plotted as a function of libration angle in Fig. 4 for seven Trojans, showing a close link between libration amplitude and the extent of semi-major axis variation during libration.

2.2 Population Studies

There may be a difference in the number of objects librating around each Lagrange point (Fig. 5a) since approximately three fifths of the currently accepted Jupiter Trojans are known to be in the L_4 swarm. More specifically, as of February 2006, 1120 L_4 and 747 L_5 Trojans were known, 54 new L_4, and 87 new L_5 objects having been discovered since February 2005. Shoemaker et al. (1989) pointed out that bright Trojans were equally numerous in the L_4 and L_5 swarms, but attributed the presence of more dim Trojans at L_4, to more numerous collisions there. Milani (1993) conducted a study on families in the Trojan swarms, groups of objects with similar characteristics which are possible fragments of collisions, and found that the L_5 point was lacking in significant families while the L_4 swarm contained approximately four three-or-more groupings and 3 couples.

In contrast, the L_5 swarm contained only one triplet and no significant couples. It remains unclear whether the differences between the swarms are of observational origin or are due to another mechanism, and the point is not further discussed here.

Recent attempts have been made to explain the absolute magnitude distributions of Trojans and their breaks in slope (Jewitt et al., 2000; Lagerkvist et al., 2002; Yoshida and Nakamura, 2005). Assuming a shared visual albedo (of 0.04), the magnitude distribution may be converted to a size distribution, as detailed by Jewitt et al. (2000). From the Minor Planet Center catalogue of 8 May 2006, the absolute magnitude distribution of 1825 objects in the semi-major axis range between 4.729 AU and 5.656 AU is plotted against cumulative logarithmic frequency count in Fig. 3b. Using a least-squares fit to the absolute magnitude distribution, slope parameters m were obtained for the ranges 7-9.5 absolute magnitude and 9.5-12.5 absolute magnitude. Assuming that the radii of Jupiter Trojans follow a power-law distribution $n(r) = Ir^{-q}dr$ (I being a constant) such that n objects are within the radius range dr, q is related to the calculated slope parameters m by the relation $q = 5m + 1$ (Jewitt et al., 2000). For absolute magnitude < 9.5, q is calculated as 5.315, and between 9.5 and 12.5, q is 2.98, (Fig. 5b). These agree within error with Jewitt et al.'s (2000) calculation of 5.5 +/- 0.9 and 3.0 +/- 0.3 for absolute magnitude for the same respective magnitude ranges, which applied to the L_4 cloud. We can confirm the slopes and break in the size distribution using present improved catalog data rather than requiring a large telescope to do a survey. However, the second break at absolute magnitude of approximately 16, claimed by Yoshida and Nakamura (2005) in a deep survey of L_4 using the 8-m Subaru telescope, lies beyond the completeness limit of the catalogs used here, and we are unable to confirm it. According to Marzari et al. (2002), the different slopes correspond to distinct populations of large (over 30-45 km in radius) and small Trojans (under 30-45 km in radius), the large-size population being assumed to represent leftover material from the formation of Jupiter, while the small-size population is collisional fragments. Since we now confirm the result of Jewitt et al. (2000) regarding the slope of the Trojan population, we repeat Marzari's assertion that this slope would mean that there are more Trojans than Main Belt asteroids.

2.3 Dynamical Studies

While epicyclic motion arises even in the case of Keplerian motion (see e.g. Mikkola et al., 2006), the longer period libration can be regarded to first order as originating through the motion of the co-orbital body in a potential well (Érdi, 1997). If α_0 is the average longitude of the object from Jupiter (i.e. 60^0) and the full extent of libration is from α_0-α to $\alpha_0+\alpha$ then α is the amplitude of libration. The amplitude of libration can be regarded as one of the proper elements of Trojan motion (Beauge and Roig, 2001). That is to say, it does not change in the short term, although its secular change may be of interest. To determine the librational behavior, we used numerical integration in a realistic solar system numerical model. We also studied the long-term evolution of Trojans in this manner. The Mercury integration package of Chambers (1999) was used, with the Sun and all the planets used for the integrations.

3. STATISTICAL PROPERTIES

Some previous studies attempted to find relationships among parameters of the Trojans (Schubart and Bien, 1987). Such studies often benefit from removal of short-term perturbations which characterize the osculating orbits. Averaging theories generally produce proper elements which are close to invariants of the motion. In the case of Trojans, heavily locked into resonance, there are three proper elements, the proper eccentricity, the proper inclination, and the amplitude of libration (Milani, 1993). These have been used among other things to identify possible families, likely of collisional origin, among the Trojans. Apart from noting that as in the main belt, families imply collisions and thus could be a source of horseshoe objects, we do not further discuss them.

3.1 Érdi Theory

In the course of our integrations of many Trojan orbits, and with our aim to find horseshoe objects, we focused on amplitude of libration. Here we present results that may be compared to the predictions of a theory of the motion of Trojans going beyond the restricted three-body problem (Érdi, 1978).

In Trojan libration around L_4, Érdi (1978) considered the time-averaged longitude difference (in Jupiter's orbital plane) from Jupiter to be α_0 and found an expansion giving this as a function of an expansion parameter l:

$$
\begin{aligned}
\alpha_0 = {} & \frac{\pi}{3} + \frac{3\sqrt{3}}{2^3} l^2 + \frac{13\sqrt{3}}{2^8} l^4 + l\cos\varphi \\
& - \left(\frac{\sqrt{3}}{2^3} l^2 + \frac{\sqrt{3}}{2^8 3^2} l^4 \right) \cos 2\varphi + \left(\frac{5}{2^6} l^3 - \frac{65}{2^{12}} l^5 \right) \cos 3\varphi \qquad (1) \\
& - \frac{25\sqrt{3}}{2^7 3^2} l^4 \cos 4\varphi + \frac{1283}{2^{12} 3 \cdot 5} l^5 \cos 5\varphi + O(l^6)
\end{aligned}
$$

with $\varphi = \sqrt{\frac{27}{4}\left(1 - \frac{3}{8} l^2 - \frac{97}{512} l^4\right)} u + \delta$, where u is an angular expansion parameter and δ a constant of integration. A parameter related to the mass ratio μ often arises in the three-body problem and in this case expansions were done in terms of $\varepsilon = \sqrt{\mu}$, with $u = \varepsilon(\upsilon - \upsilon^0)$, where υ is the true anomaly of Jupiter. To find the limits of the longitude difference, and thus the amplitude of libration, we used values of 0 and π for l. Érdi found the period of libration to be given by

$$
T_l = \frac{T_J}{\varepsilon \sqrt{\frac{27}{4}\left(1 - \frac{3}{8} l^2 - \frac{97}{512} l^4\right)}} \qquad (2)
$$

where T_J is the orbital period of Jupiter. He used T_J = 11.862 years and ε = 0.030885, compatible with modern values. Our computations reproduced values given in his paper to more than the significance given. We compared the values obtained from our orbital integrations to the theory. We made time series of the angular separation of each Trojan studied from Jupiter, used a simple algorithm to find the extrema of this series, and found the periods of libration by inspection of their period of repetition.

3.2 Observational Data

Figure 6a shows the amplitudes of libration plotted as a function of libration period for 1510 accepted Trojan objects as in the Minor Planet Centre Orbital Database. The general trend is that the libration period increases with libration amplitude, a result found for considerably fewer (i.e. 40) objects by Bien and Schubart (1987). We have overplotted the theoretical result described above, based on the theory of Érdi (1978, 1997). Since Érdi's theory is for objects coplanar with Jupiter, that is with inclination zero, we have graphed asteroids with inclination less than 7º separately (Fig. 6b). It can be seen that Érdi's results match very well with the results of our orbital calculations, for those objects with low inclination. Since for the same amplitude of libration, most objects of large inclination have longer libration periods than objects of low inclination, libration period must be an increasing function of inclination.

Since inclination was not incorporated into Érdi 's (1997) theory, an attempt was made to correct for it. The amplitude of libration given by theory was taken as the projection onto Jupiter's orbital plane. For inclined orbits, this projection was used to calculate the actual amplitude of libration extending out of Jupiter's orbital plane. For this, C is the fraction of the orbit of Jupiter the object traverses in Jupiter's corotating frame when projected onto Jupiter's orbital plane. The equation *C=rd* comes from equating the fraction of circumference to the fraction of angle transcribed by the object in Jupiter's corotating frame, where *r* is the radius, and *d* is the measured amplitude of libration. The extent of motion in and out of Jupiter's plane (i.e. in the *z*-direction) is given by *z*. This can be used roughly to correct for inclined objects in Érdi (1997), as the motion in and out of Jupiter's plane acts within a short timescale of approximately the orbital period of the object, this motion is averaged away over the longer timescale of the period of libration. Our result is that the corrected amplitude of libration *d'* is

$$d' = \frac{\sqrt{C^2 + \left(\frac{1}{2}z\right)^2}}{r} \quad (3).$$

The results of applying this correction can be seen in Fig. 6c.

In a survey of the L4 Trojan cloud Lagerkvist et al. (2002) observed 399 moving objects classified as Trojans. Using just numbered and multiopposition asteroids as of 2002, they note the average librational amplitude in the L_4 and L_5 clouds as 15.45^0 and 14.45^0, respectively. Using the extended list of 1510 objects, we calculated the average librational amplitude in the L_4 and L_5 clouds as 15.4^0 and 14.6^0, respectively. The maximum librational amplitudes are 54.5^0 (2002 GY_{162}) in L_4 and 48.8^0 (1998 MV_{47}) in L_5. An amplitude of libration histogram was constructed and plotted with one based on the proper elements of Milani and Knezevic found in the Astdys website (hamilton.dm.unipi.it/cgi-bin/astdys/astibo), using 1510 objects (Fig. 7). These histograms are very similar, suggesting that our amplitudes of libration and those derived from the theory of Beauge and Roig (2001) as computed by Milani and Knezevic are in agreement. This may be taken as a computational verification of the correctness of the Beauge and Roig theory.

Érdi's (1984) theory predicts the relationship between Trojan nodal regression and amplitude of libration. The general trend of this relationship is for the magnitude of nodal regression to increase with amplitude of libration, which can be seen in Fig. 7 both for calculated osculating elements and for Milani and Knezevic elements. For both data sets the nodal motion is strongly biased negatively with respect to that of Érdi's (1984) theory, which only incorporates a three body model and not the perturbing effects from other planets. We further note that both histograms of number of objects as a function of amplitude of libration show minor gaps just above 10º and just above 15º. We do not find a convincing relationship to rate of regression although we note that Jupiter's nodal regression rate intersects the distribution approximately where these gaps occur.

An outgrowth of Érdi's theory (1984) did incorporate inclination, in particular its effect on nodal rates. To test this, we used the Milani and Knezevic elements to plot nodal rates as a function of the inclination. Fig 8 shows this relation, with dependence actually on *sin* of the proper inclination ($sin\ I_p$). To facilitate comparison to the results of Érdi (1984), points have been grouped amplitude of libration to show that not only is the general form of the relation similar to his, i.e. a slowly rising curve involving the square of $sin\ I_p$, but also the dependence on amplitude of libration similar to what he found. However, as before we note that the nodal motion is heavily biased negatively. In the context of Érdi's (1984) discussion of a critical inclination beyond which Trojans should have a positive rate of nodal motion, we find that almost none do. His modified three body problem result suggested that many low-libration amplitude objects should have a positive rate. For the lowest libration amplitude objects, we note a transition of rate of motion at about the same inclination as Érdi's (1984) theory suggested for those with the highest (in that case roughly 20 degrees) amplitude.

Overall these results suggest that a three body framework is suitable for discussing some aspects of Trojan motion, such as the relations between amplitude of libration and period of libration. The rate of nodal motion in particular is not well explained in this framework. Morais (1999, 2001) has developed a more complete Hamiltonian-based theory of Trojan motion which can include the effects of planetary oblateness and other perturbers. A more complete theory is likely required to accurately model the relationship between libration amplitude and nodal regression. We suggest exploration of the applicability of Morais' theory through comparison to observational parameters would be a worthwhile exercise.

4. Type of Orbits

Our survey of the orbits of Jupiter Trojan swarms was performed based on data from the Minor Planet Center. Using the Mercury integrator package we integrated all known and suspected Jupiter Trojans for approximately ten thousand years forward from the current date, in an effort to catalogue the motions currently exhibited by the objects. We initially focused on Trojans with extreme *a*, finding no horseshoe objects, and then tested the whole set of integrated objects by finding libration amplitudes around each of L_4 and L_5 as described above.

4.1 Absence of Horseshoe Librators

It is to be expected that the Jupiter Trojan population should have a smaller fraction of horseshoe objects than other Trojan populations. The higher mass ratio of Jupiter and the Sun than any other planet and the sun presents the most unstable three-body scenario for horseshoe objects. Dermott and Murray (1981) note that horseshoe objects for a low mass ratio between the Sun and the host planet are stable over much longer timescales than for higher mass ratios. This is because the high mass of the planet amplifies the close encounter effects between the planet and the horseshoe librator. The lifetime is given by $\Gamma \leq \frac{T}{\mu^{\frac{5}{3}}}$, where T is the orbital period. For Jupiter this value is only 10^6 years and Dermott and Murray stated the expectation that there should be no horseshoe librators. We confirm this expectation by having determined that there are no known Jupiter-associated objects with well-defined orbits exhibiting horseshoe behaviour, among 1618 objects examined. Further deep surveys would be useful, as the domain where collisional fragments may be injected into horseshoe orbits could then be explored and a timescale for collisions inferred. We note that if one integrates short-arc objects, they often appear to have a horseshoe orbit. We do not have a precise explanation of why poorly defined orbits should often appear to be horseshoes, but merely note this fact and caution that well-determined orbits are essential in such studies.

4.2 Most Trojan-like non-Trojan

The asteroid 118624 (2000 HR_{24}) appears to be circulating rather than in 1:1 resonance despite being within the limits within which resonant behaviour is possible. This object was one of the only Jupiter Trojans which was exhibited behaviour warranting further investigations, as well as having a well-defined orbit. It had a noticeably smaller semi-major axis than most of the other Trojan candidates; as well, its current x-y position shows it to be on the other side of the Sun from Jupiter, placing it outside of the main swarms (Fig. 2), indicating possible horseshoe motion. However, further integrations and a clone study showed it to be simply circulating and not in fact in any kind of resonance with Jupiter, a result already noted by Beauge and Roig (2001). It is possible that this object is in fact a recently-escaped Jupiter Trojan from one of the swarms, however this is not hinted at by the clone study: as can be seen from the graph, it is more likely that 2000 HR_{24} approximately one thousand years ago had a semi major axis of about 4.5 AU, slightly less than its current value (Fig. 9). This can be deduced from the density of the traces near that time. However it is clear that the traces diverge about 600 years back and 200 years in the future, and that there are regular interactions with Jupiter between then and now. Based on current uncertainties in the orbit and due to these strong interactions, whose details depend critically on distance to Jupiter, it is clear that we cannot trace the orbit with certainty beyond this 800 year window. This is a good example of chaos in action and prevents us knowing much about the origins of 2000 HR_{24}.

5. DISCUSSION

Our survey of the Jupiter Trojan swarms' current motions did not reveal any horseshoe librators. Those objects boosted into a horseshoe orbit by collisions should have a short lifetime. The presence of families, although not discussed in detail in the present work, suggests that collisions do take place in the Trojan clouds but likely not frequently.

The indication from the clone study of 2000 HR_{24} that this object is most likely not an escaped Jupiter Trojan is reasonable as the Jupiter Trojan swarms are relatively stable over the lifetime of a circulating object like 2000 HR_{24} (approximately a few hundred to a few thousand years). Although_unlikely, it is possible for a fair number of objects in the solar system to be escaped Trojans, as the swarms have been shown to be in a state of flux as a “dynamically unstable structure” (Levison et al. 1997). It has been estimated by Levison et al. (1997) that there are currently more than 200 evaporated Jupiter Trojans with diameters greater than one kilometer traveling the solar system. However we do not find any intermediate objects, as horseshoe objects might be expected to be.

5. SUMMARY

We to performed a large scale survey of the approximately 1,600 known and suspected Jupiter Trojans in search of possible horseshoe librators. It was expected that a small number could be undergoing horseshoe motion, due collisions or evaporation of the Trojan swarms. However, the only orbits catalogued were tadpole orbits, as well as one shown to be simply circulating, 2000 HR_{24}. Deeper surveys may yet reveal horseshoe objects. It is interesting to note that Brown (1911, 1912) had suggested surveys for horseshoe objects at their stationary points 23.4º from Jupiter when only four Trojans were known and this idea may yet guide searches.

ACKNOWLEDGEMENTS

This work was supported by the Academic Research Fund of Athabasca University. RGS thanks NSERC for an Undergraduate Summer Research Award. We also thank D. P.

Hube for use of computers and office space. We thank Seppo Mikkola for use of a clone generation program which was passed on through Ramon Brasser. We would also like to thank Ramon Brasser for invaluable advice and help along the way. Paul Wiegert has also given useful comments on some aspects of this study and furnished Fig. 1. Use of the NASA Astrophyisics Data System, including for study of historic documents, was very helpful in preparing this work.

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Figure Captions.

Fig. 1. Some examples of zero-velocity curves associated with tadpole and horseshoe orbits, as viewed in a reference frame that co-rotates with the planet (the scale has been adjusted so that the planet is one unit from the Sun). The respective names of orbits are obtained from their characteristic shapes. The L_4 and L_5 points are each encircled by 'tadpole' or Trojan-like curves for which a typical zero-velocity curve is shown. Both the L_4 and L_5 points are encompassed by a 'horseshoe' curve. The width of the orbital regions in the radial direction has been greatly exaggerated for clarity. The L_4 point leads Jupiter in its (counterclockwise) orbit around the Sun, while the L_5 point trails it.

Fig. 2. Views of the distribution of the 1618 objects classified as Jupiter Trojans as of May 2003, depicted in their positions on JD 2451000.5 (July 6, 1998). The top frame

shows the view from above the ecliptic plane, with the Sun at centre, and Jupiter indicated. The L_4 Lagrange point is at right, and all objects move in counterclockwise fashion about the Sun. The L_5 Lagrange point is at the left. Asteroid 2000 HR_{24} is labeled far from the Lagrange points, and discussed further in the text. The bottom panel shows the view looking in past Jupiter toward the Sun and illustrates the rather large vertical extent of the Trojan clouds, associated with the generally large inclinations of the Trojan asteroids. Jupiter and the Sun are not to scale relative to each other nor to the scale of the solar system. The Lagrange points are 5.2 AU from the Sun.

Fig. 3 Six characteristic tadpole orbits depicted over one approximate libration period (~160 years) in Jupiter's co rotating frame, centred on the sun. Broken into three frames for clarity of the orbits. Jupiter, on the right of each frame, is at approximately 5.2 AU. The asteroids oscillate about the Lagrange points which are located at 5.2 AU and either 60° in front of or behind Jupiter. Note the longer term elongated libration superimposed over the shorter term loops. 1996 RX15 is on the upper end of both angular libration amplitude as well as libration period for the Jupiter Trojan population, while 1973 SB_2 has low libration amplitude and remains near the L_4 point. Jupiter and the Sun are not to scale relative to each other nor to the scale of the solar system.

Fig. 4 A plot of difference in semi-major axis and difference in mean longitude (approximately the difference in angular separation) between Jupiter and seven characteristic tadpole orbits over one libration period. As can be seen, four objects correspond to the L4 point (60° in front of Jupiter) and three objects correspond to the L5 point (60° trailing Jupiter).

Fig. 5 Cumulative magnitude distributions of Trojan asteroids, giving an indirect look at their size distribution. (a) These histograms of the absolute magnitude of Jupiter Trojans were created using accepted objects as of May 2003. The discrepancy in the number of objects between the L4 swarm and the L5 swarm is apparent. The tapering off for higher magnitudes indicates when the survey becomes incomplete. Since this starts for brighter objects in the L5 swarm, there is an indication that the discrepancy may be at least in part due to observational differences. (b) Logarithmic cumulative histogram using accepted objects from both swarms as of May 2006. Linear fits are possible in logarithmic space and a break in the distribution (see text) is seen for magnitude 9.5. The break in slope at magnitude 12.5 indicates incompleteness of the survey.

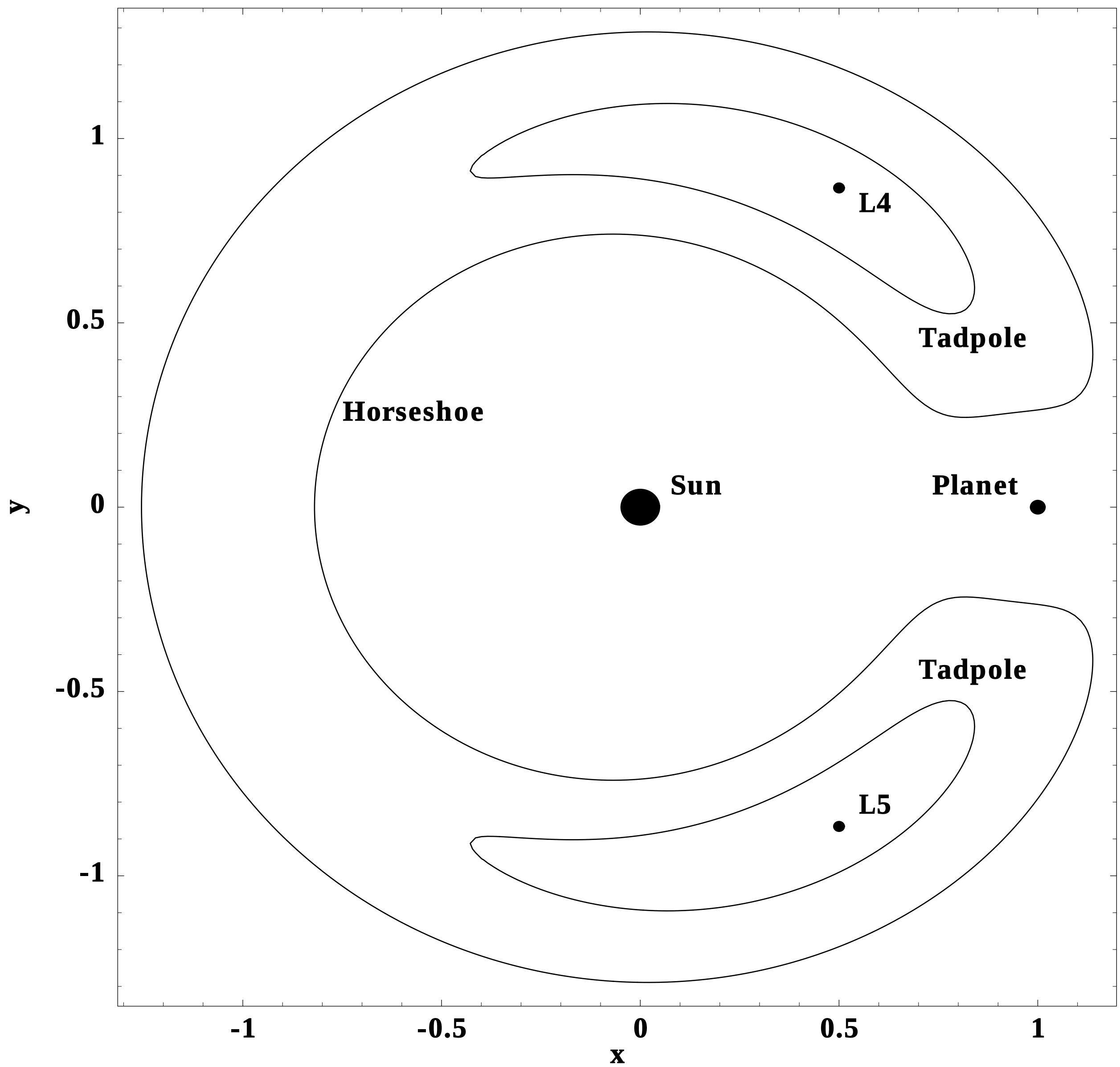
Fig. 6 (a) A plot of period of libration versus amplitude of libration for 1584 accepted Jupiter Trojans. Libration amplitude and period were calculated over one libration period beginning approximately JD 2451000.5 (July 6, 1998). 118 objects were omitted due to complications in obtaining period of libration and subsequently amplitude of libration. It should be noted that these omitted objects tended to be on the extreme low end of libration amplitude. The theoretical curve crosses the period of libration axis at ~147 years, which agrees with theory (Erdi, 1997). (b) The same plot as above, using only the 449 objects with attainable libration period and amplitude with inclination under 7°. (c)

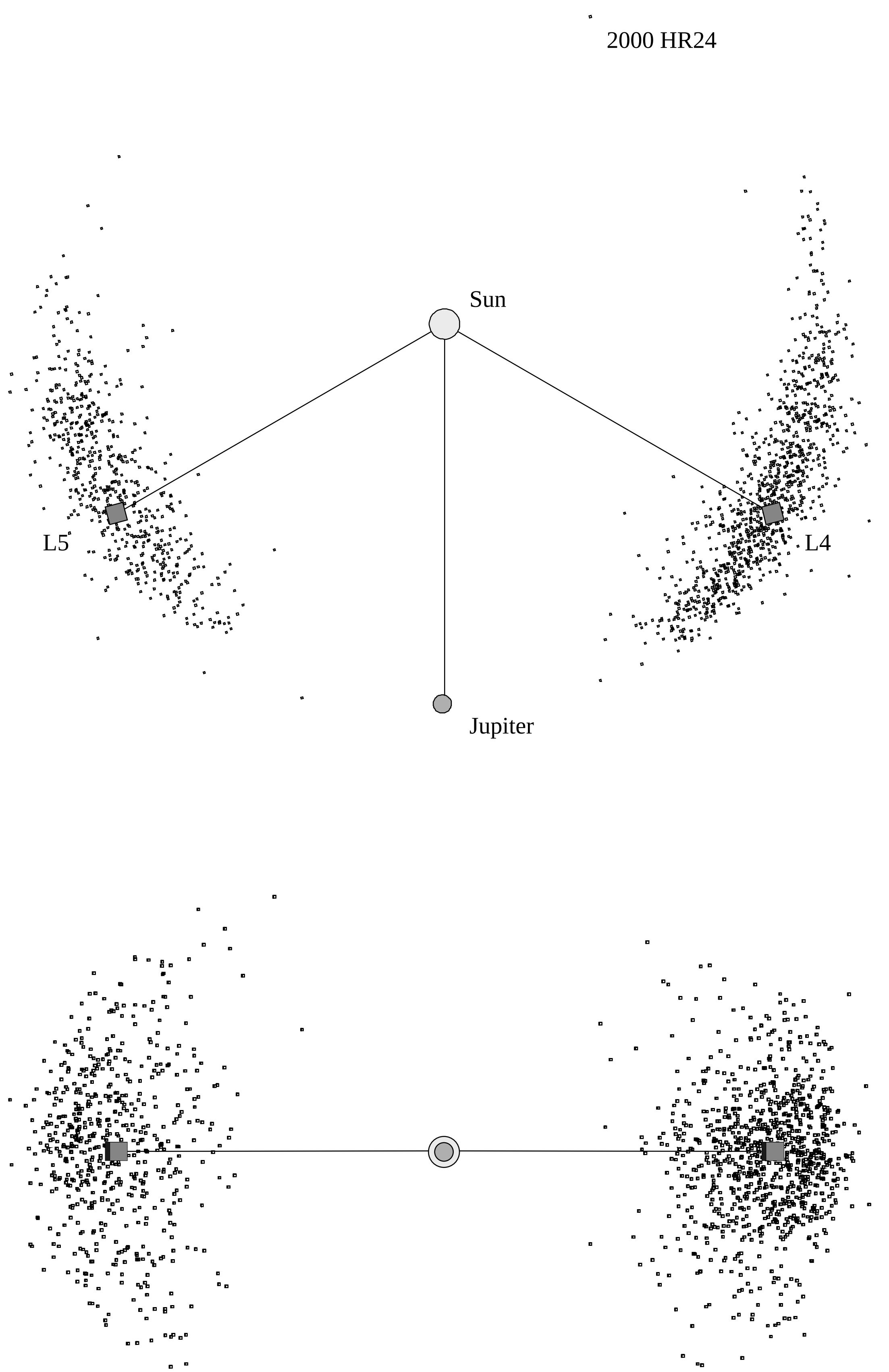
Period of libration plotted against corrected amplitude of libration, as per equation (3), using the objects from (a). Note the stronger correlation to theory than for uncorrected inclined objects.

Fig 7 Erdi's (1984) theoretical curve is plotted overtop two data sets for amplitude of libration versus regression rate. The left hand panel uses amplitude of libration calculated over one libration period and using osculating orbits, and regression rate calculated over a three thousand year period. The right hand panel uses Milani and Knezevic proper elements. The osculating elements are more scattered than the proper elements, which is to be expected. Erdi's theory matches the trend well, but is obviously offset vertically from the data. Accompanying histograms confirm the similarity between the two data sets. Gaps in the amplitude of libration distribution possibly correspond to Jupiter's precession rate (horizontal line).

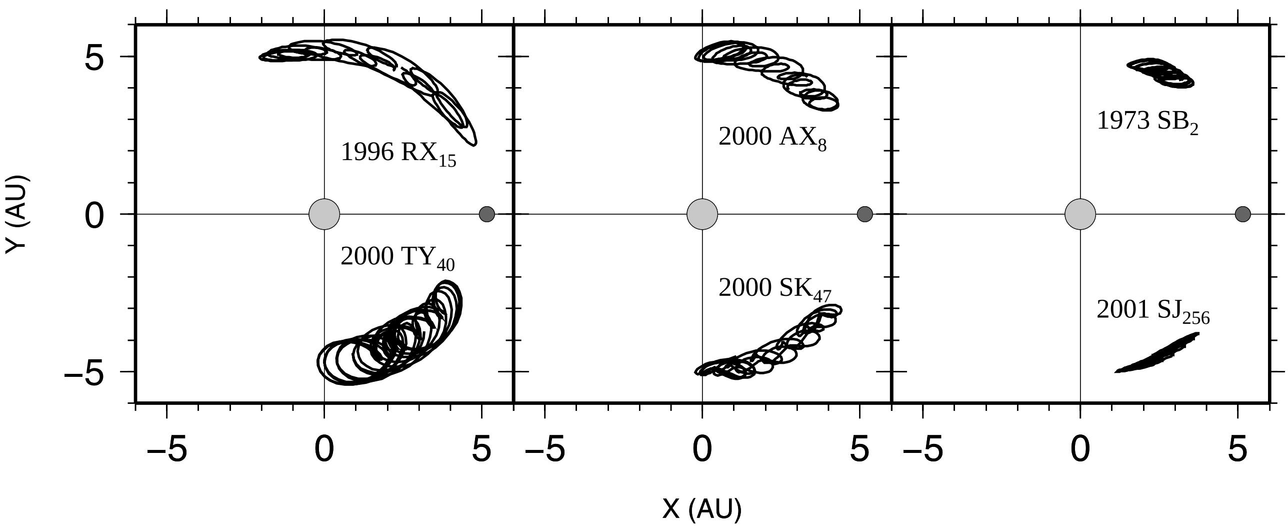
Fig 8 Trojan nodal rates as a function of sin of proper inclination. The sets of points correspond to amplitudes of libration (from top down) of 0-5 degrees, 19-21 degrees, 29-31 degrees, and (few points) 35-45 degrees, and the size of the dot is proportional to the amplitude of libration. The trend of the curves is similar to that derived by Erdi (1984) but with considerable negative bias.

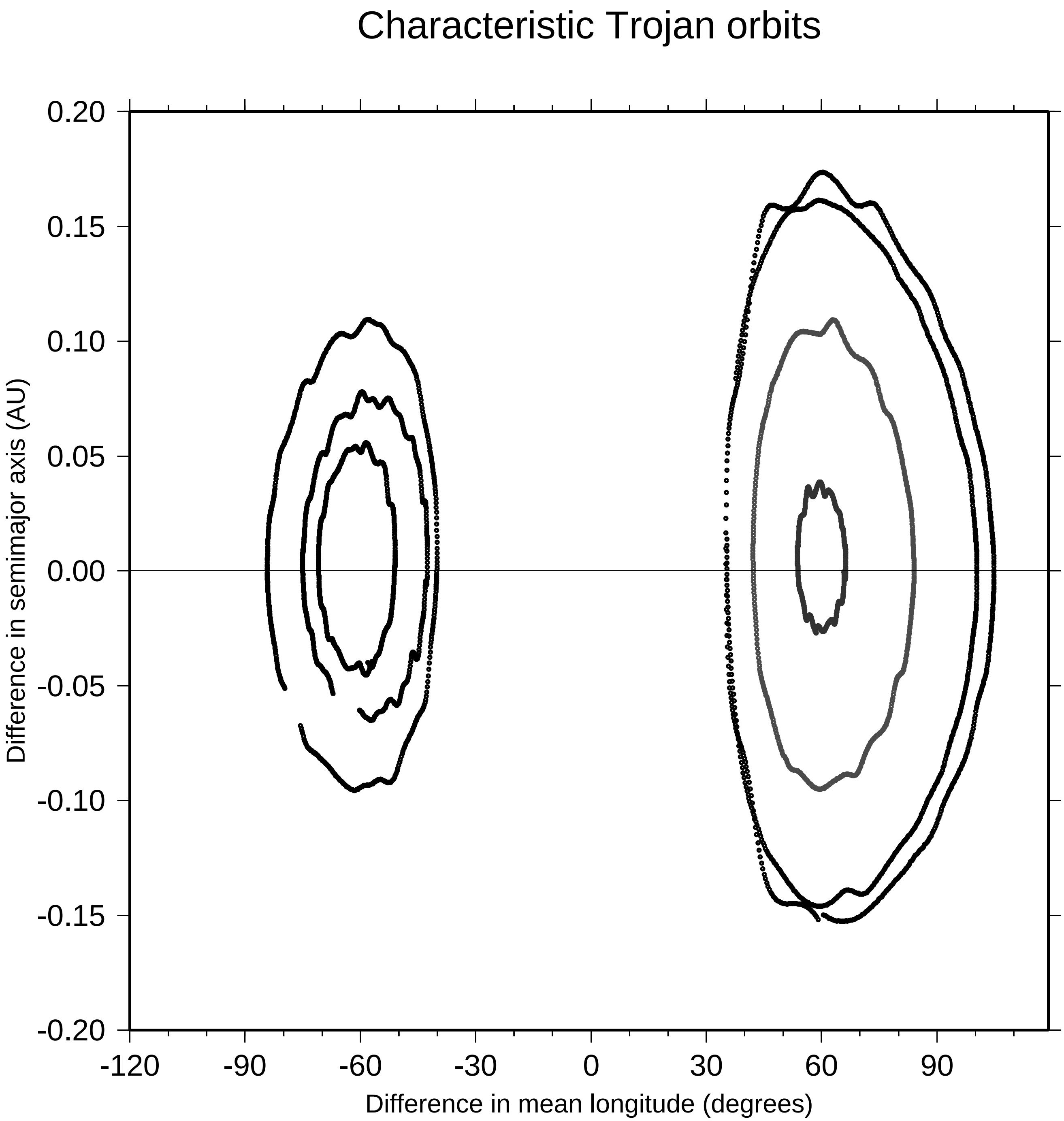
Fig. 9 2000 HR24 is known to be circulating with respect to Jupiter at the present date. In an attempt to establish whether this object is an escaped Jupiter Trojan or not fifty clones were integrated backwards from the present date for 1000 years. It and fifty clones were also integrated forward to examine the possibility of becoming a Jupiter Trojan. A decisive conclusion cannot be reached as to its origins (due to what is likely a close approach with Jupiter at ~1500 A.D.) although the density of the traces suggest that as of ~1000 A.D. 2000 HR24 had a semi-major axis between 4.1 AU and 4.9 AU. It is kicked out of its periodic motion by another close approach with Jupiter in a few hundred years. This places it outside of the Jupiter Trojan swarms. Due to the chaos demonstrated here, integrations going further back or forward in time are not likely to be useful.


1
0.5
0
-0.5
-1
y
-1
-0.5
0
0.5
1
x
L4
Tadpole
Horseshoe
Sun
Planet
Tadpole
L5


2000 HR24
Sun
L5
L4
Jupiter

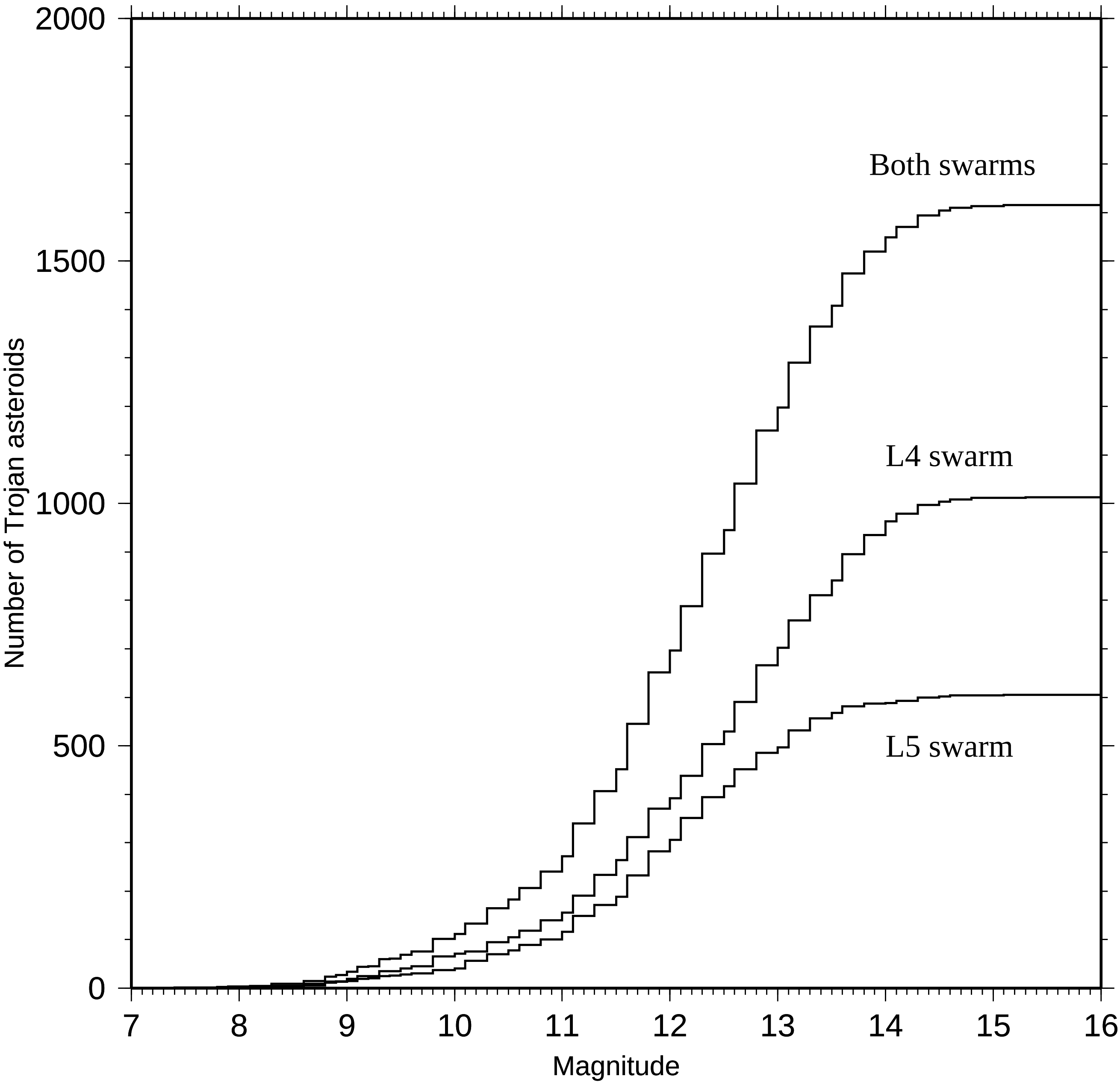
Characteristic Trojan orbits, in Jupiter's co-rotating frame

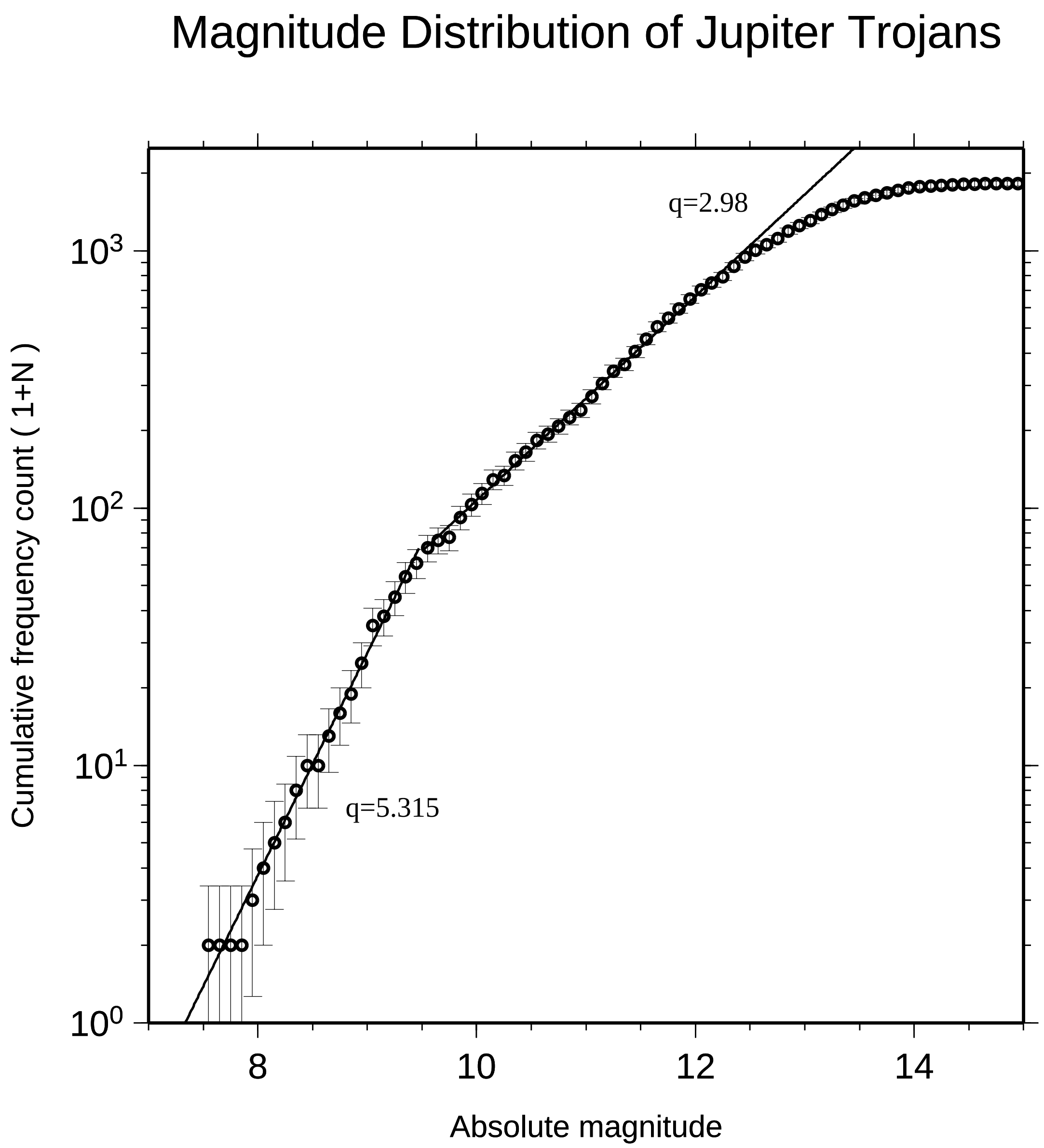




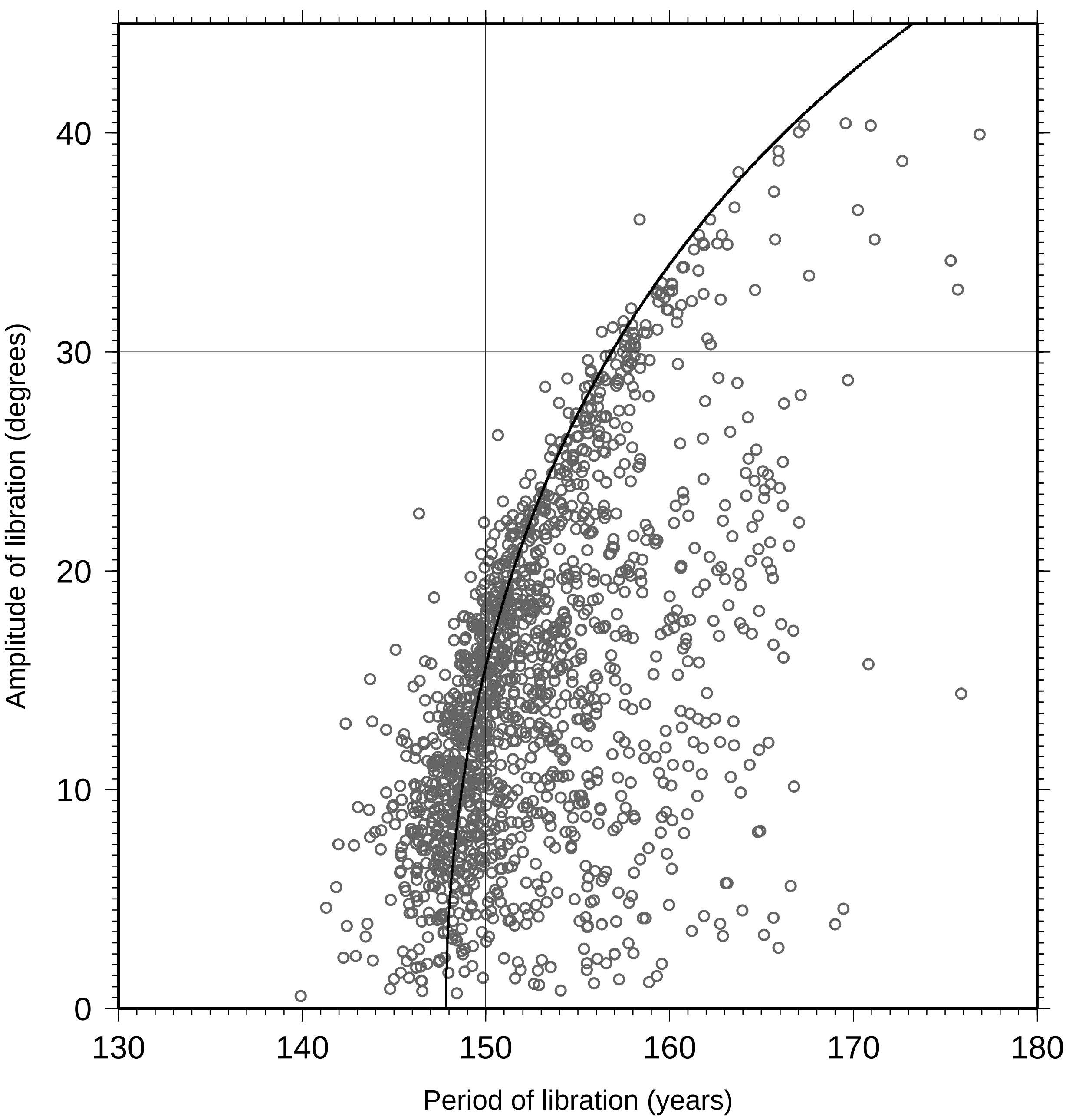
Characteristic Trojan orbits
0.20
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
-0.20
Difference in semimajor axis (AU)
-120
-90
-60
-30
0
30
60
90
Difference in mean longitude (degrees)

Magnitude distribution of the trojan swarms

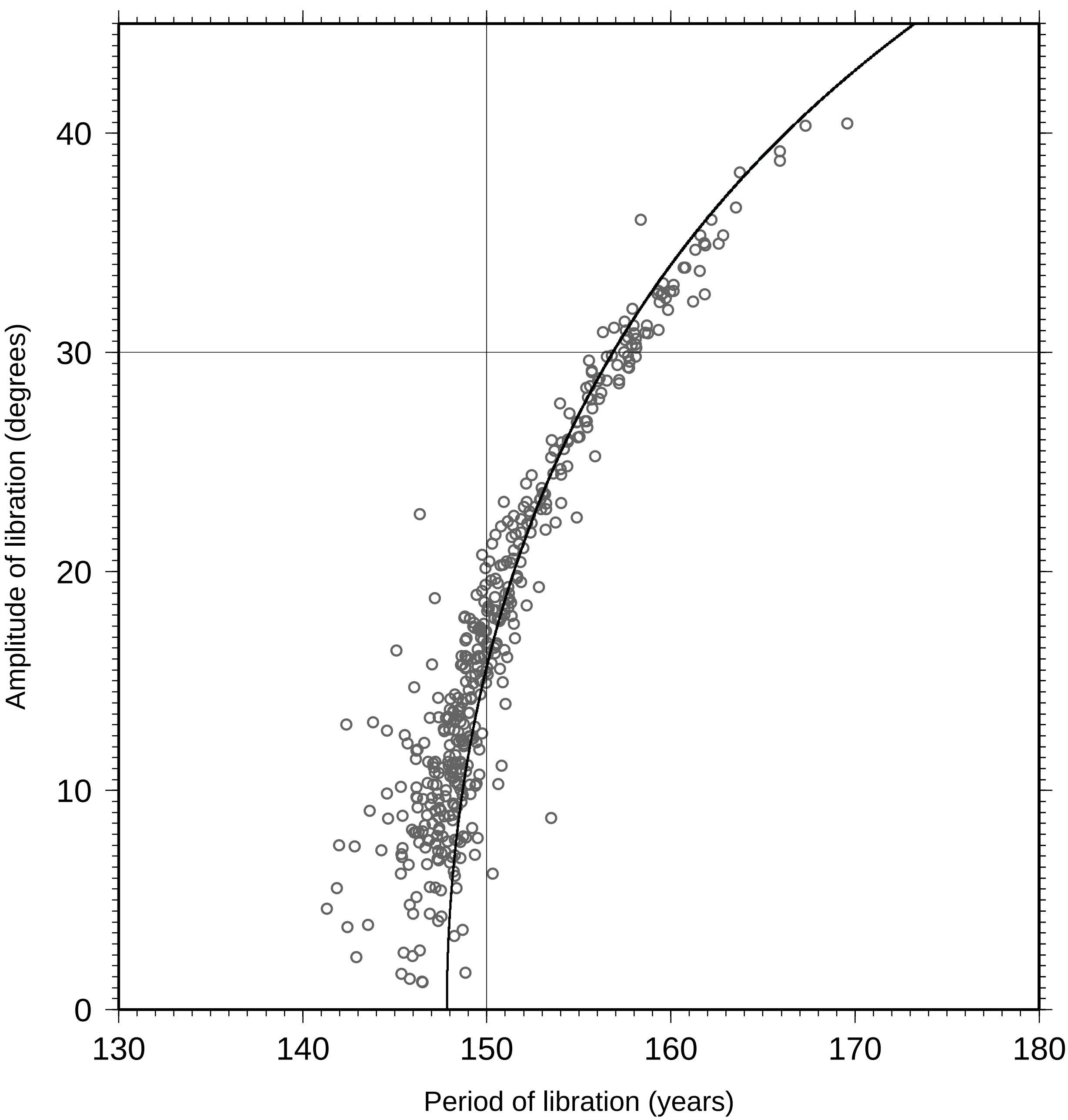



Magnitude Distribution of Jupiter Trojans
Cumulative frequency count (1+N)
Absolute magnitude
q=2.98
q=5.315
10^3
10^2
10^1
10^0
8
10
12
14

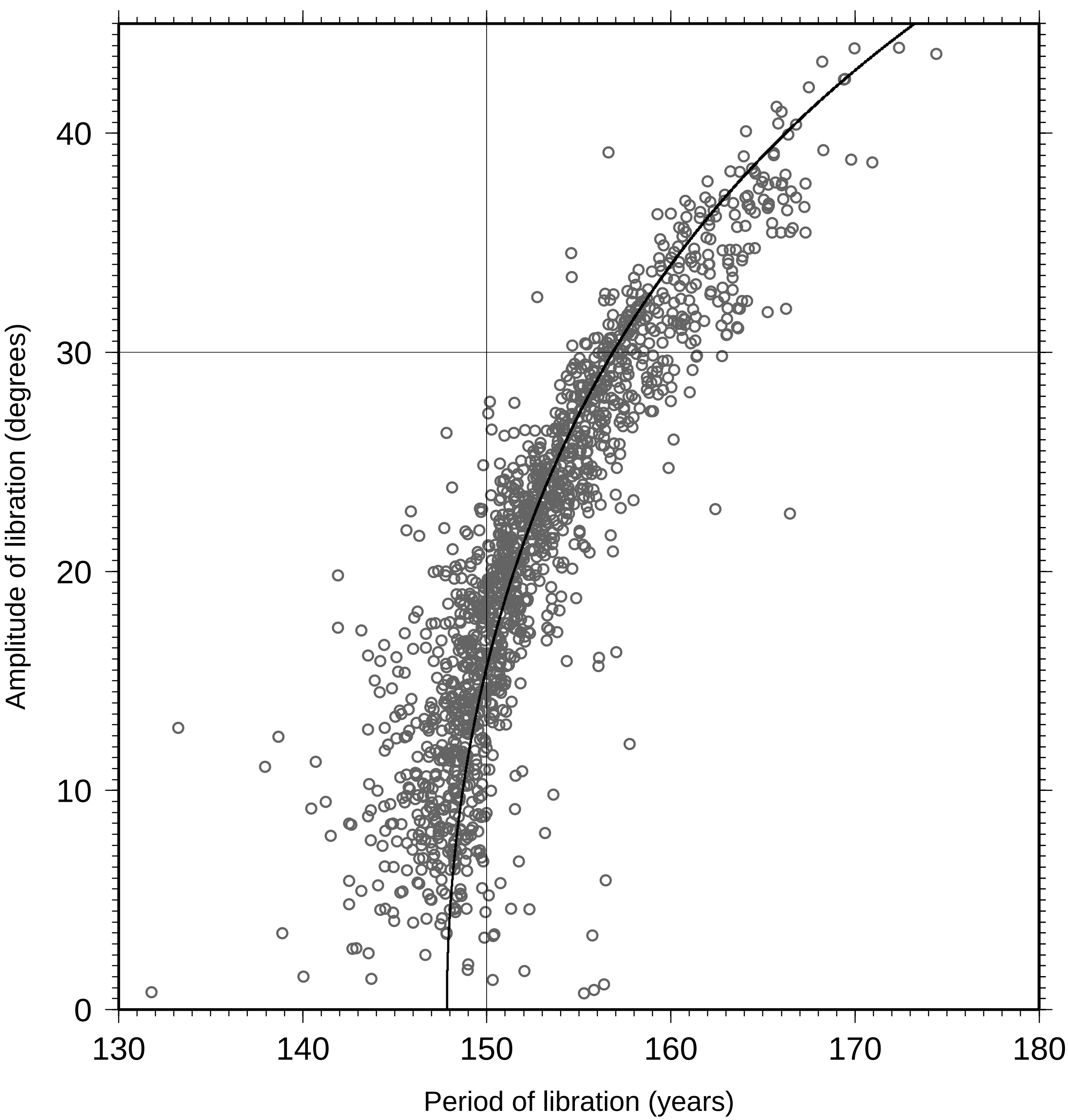
Relationship Between Libration Period and Amplitude



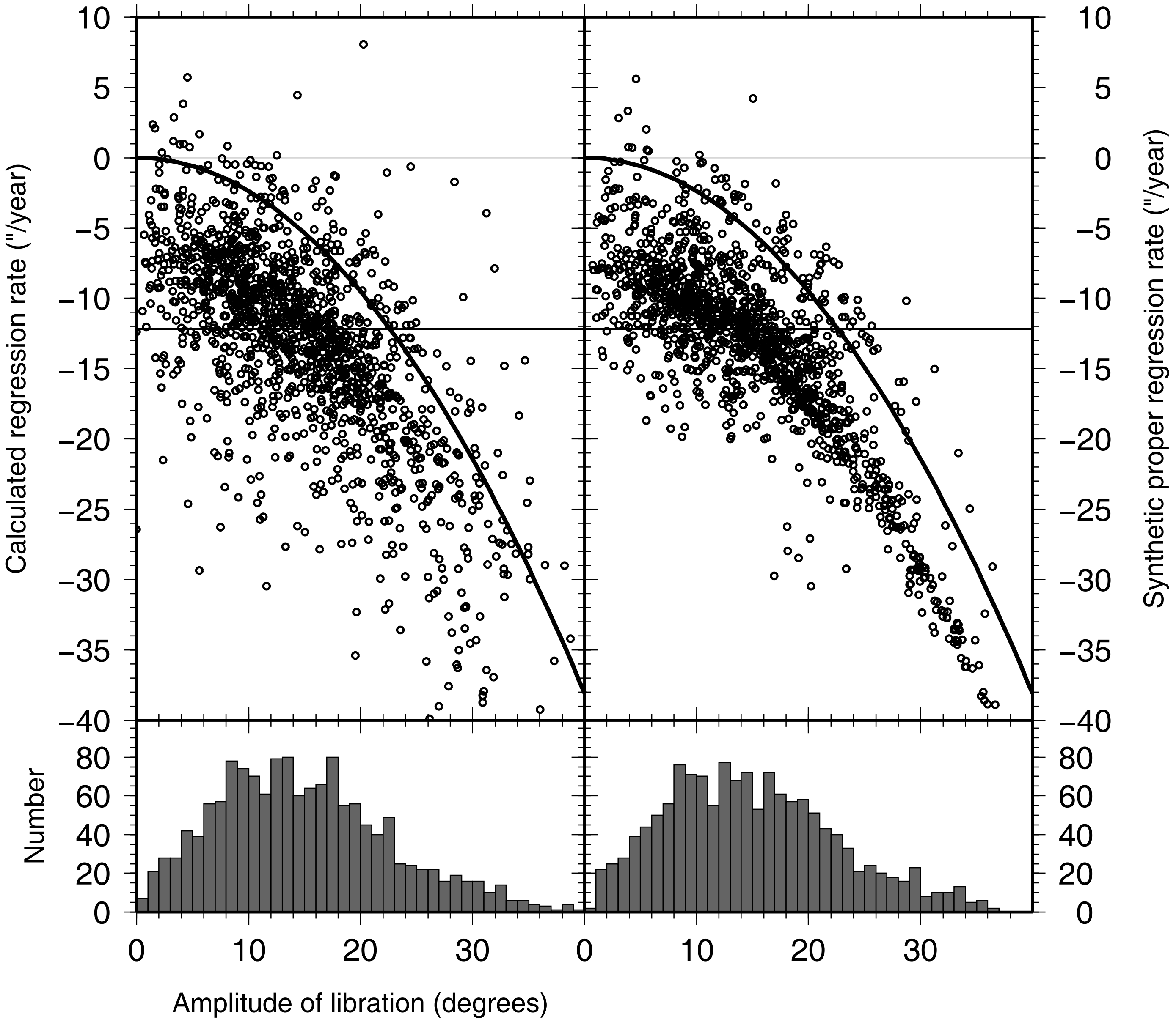
Relationship Between Libration Period and Amplitude

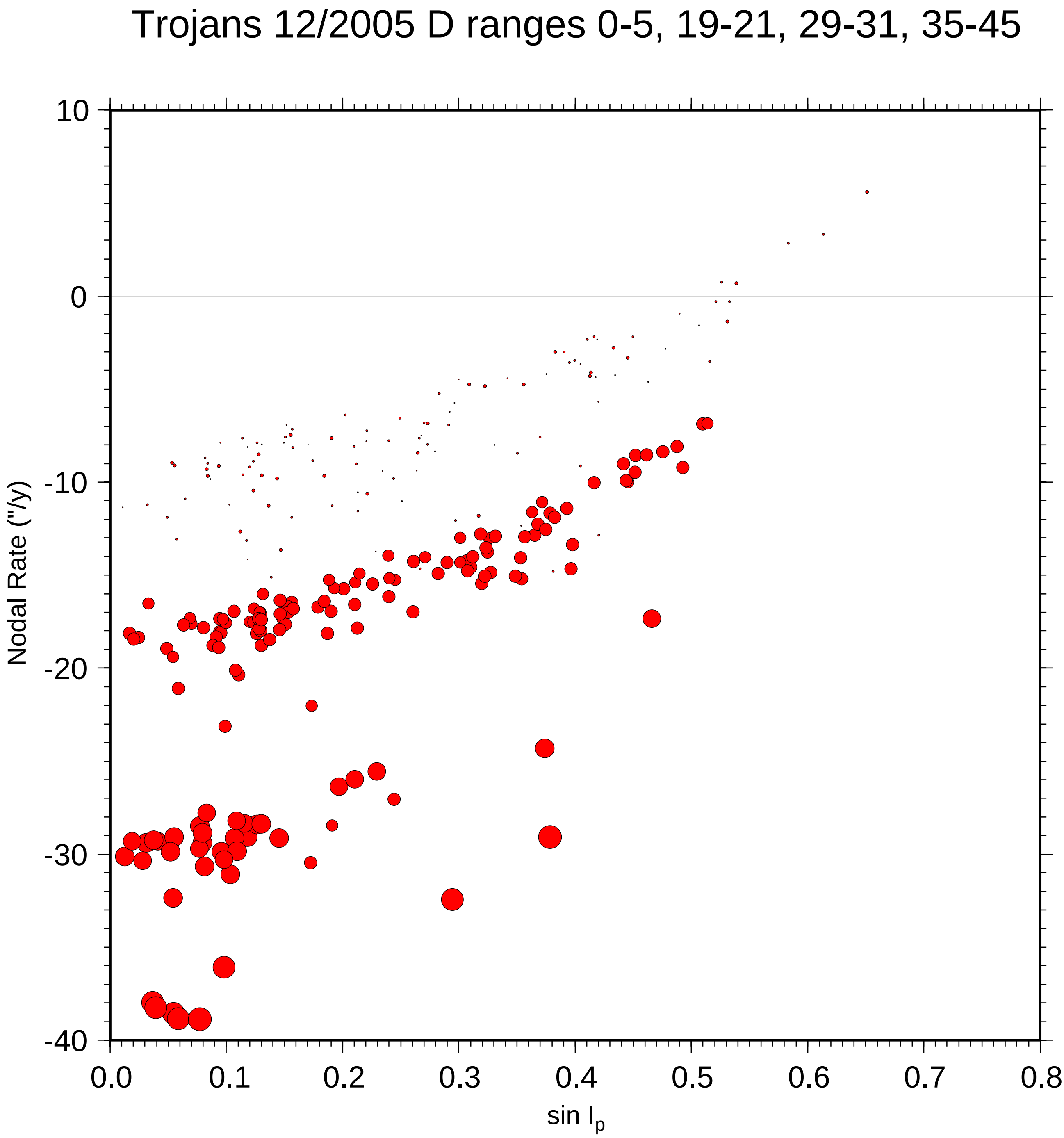


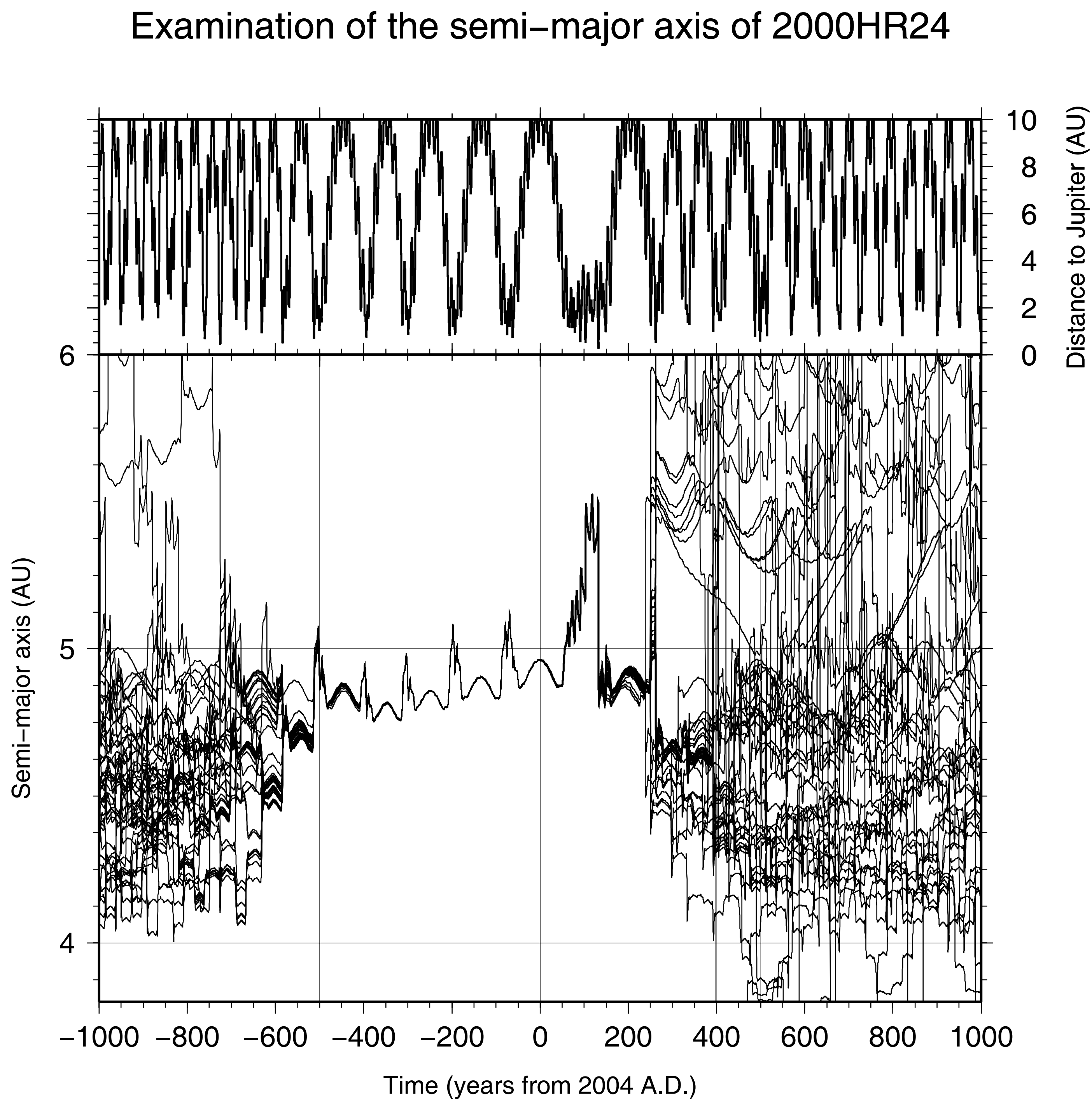
Relationship Between Libration Period and Corrected Amplitude



Amplitude of libration versus fitted node motion




Trojans 12/2005 D ranges 0-5, 19-21, 29-31, 35-45
Nodal Rate ("/y)
10
0
-10
-20
-30
-40
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
sin I$_p$



Examination of the semi−major axis of 2000HR24
Semi−major axis (AU)
Distance to Jupiter (AU)
Time (years from 2004 A.D.)
−1000
−800
−600
−400
−200
0
200
400
600
800
1000
4
5
6
0
2
4
6
8
10