Inner solar system dynamical analogs of plutinos

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ABSTRACT:

By studying orbits of asteroids potentially in 3:2 exterior mean motion resonance with Earth, Venus, and Mars, we have found plutino analogs. We identify at least 27 objects in the inner solar system dynamically protected from encounter through this resonance. These are four objects associated with Venus, six with Earth, and seventeen with Mars. Bodies in the 3:2 exterior resonance (including those in the plutino resonance associated with Neptune) orbit the Sun twice for every three orbits of the associated planet, in such a way that with sufficiently low libration amplitude close approaches to the planet are impossible. As many as 15% of Kuiper Belt objects share the 3:2 resonance, but are poorly observed. One of several resonance sweeping mechanisms during planetary migration is likely needed to explain the origin and properties of 3:2 resonant Kuiper Belt objects. Such a mechanism likely did not operate in the inner solar system. We suggest that scattering by the next planet out allows entry to, and exit from, 3:2 resonance for objects associated with Venus or Earth. 3:2 resonators of Mars, on the other hand, do not cross the paths of other planets, and have a long lifetime. There may exist some objects trapped in the 3:2 Mars resonance which are primordial, with our tests on the most promising objects known to date indicating lifetimes of at least tens of millions of years. Identifying 3:2 resonant systems in the inner Solar System permits this resonance to be studied on shorter timescales and with better determined orbits than has been possible to date, and introduces new mechanisms for entry into the resonant configuration.

Keywords: Celestial Mechanics; Resonances, Orbital; Pluto; Asteroids, Dynamics

1. Outer solar system 3:2 exterior mean motion resonance

Pluto has a remarkable 3:2 exterior mean motion resonance with Neptune. Neptune orbits the Sun three times for each two circuits by Pluto, but Pluto avoids encounter in its eccentric orbit, never being at perihelion when Neptune is near the same longitude. This allows a stable orbit for Pluto despite its orbit crossing Neptune’s. The resonance means that Pluto’s mean motion is 2/3 that of Neptune: loosely following Gallardo (2006) we will refer to this as a 3:2N exterior mean motion resonance, with N representing Neptune. Pluto’s motion also features libration, or systematic motion, of the whole orbit with respect to Neptune, over a period of about 20000 years (Cohen and Hubbard, 1965). Similar 3:2 resonance can also be found in the inner solar system, as will be discussed in section 2. In Fig. 1(a) the motion of Pluto relative to Neptune is shown for two Pluto revolutions or three Neptune revolutions. In Fig. 1(b) asteroid 67367, in 3:2E (3:2 resonance with the Earth), is shown, and similarities can clearly be seen between its orbit relative to Earth and Pluto’s orbit relative to Neptune. 67367 will be discussed in detail in section 2.3. The paths do not close due to the libration, and the orbit of Pluto is presently near one end of its librational swing. Although Rabe (1957) noted the 3:2N resonance in 1957, it was only in 1964 (Cohen and Hubbard, 1964), 34 years after Pluto’s discovery, that the libration aspects and dynamical protection mechanism were found.

[Figure 1]

In recent years many plutinos, which share the 3:2N resonance in the outer solar system, have been discovered. The periods over which they have been observed being short compared to even one full orbit, such Kuiper Belt objects in general do not have very precisely determined orbits. In the case of Pluto, discovery in 1930 has allowed only about one third of a sidereal orbit to be observed. When corrected for observational bias (Luu and Jewitt, 2002) and using the best determined orbits, (Chiang et al., 2007) about 15% of the Kuiper Belt population in the outer solar system is in the 3:2N resonance, and so protected from being destabilized by gravitational interaction with Neptune. The librational motion of 3:2N resonators is characterized by an angular resonant argument $\boldsymbol{\sigma} = 2\boldsymbol{\lambda}_N - 3\boldsymbol{\lambda} + \boldsymbol{\varpi}$, where $\boldsymbol{\lambda}$ is the mean longitude, $\boldsymbol{\varpi}$ is the longitude of perihelion, and subscript N denotes Neptune. This resonant argument for Pluto in 3:2N is shown in Fig. 2(a), librating around 180° with a period of approximately 20000 years. In Fig. 2(b), the resonant argument of 67367 can be seen similarly librating, but with a much smaller period of 430 years.

[Figure 2]

Orbital properties of all objects investigated in this paper can be found in detail on either the AstDys or the NeoDys websites (http://hamilton.dm.unipi.it/cgi-bin/astdys/astibo and http://newton.dm.unipi.it/cgi-bin/neodys/neoibo). This information includes osculating elements, magnitudes, and arc lengths. For further information on spectral class and taxonomic information, we consulted EARN (http://earn.dlr.de).

2. Inner solar system 3:2 mean motion resonance

We study here the presence of asteroids in the inner solar system whose relation to Venus, Earth, and Mars, in turn, is remarkably similar to that 3:2N resonators to Neptune. Although the generic term “inner plutinos” has been previously used (Connors et al., 2004), we will use the designation 3:2V, 3:2E, and 3:2M resonators, for those associated with Venus, Earth, and Mars, respectively. No asteroid with a semimajor axis corresponding to 3:2 Mercury mean motion resonance was found, and discovery circumstances are not favorable due to necessary proximity to the Sun in the sky. Objects in 3:2 resonance with Mercury are in any case unlikely due to its low mass. The known objects as of MJD 54000 (Sep. 21, 2006) are shown in Table 1. Several new objects have been found since the initial report on inner solar system 3:2 librators (Connors et al., 2004) and our final check for potential objects was done in early September, 2007.

[Table 1]

An asteroid in 3:2 external mean motion resonance must have a semimajor axis *a* that is close to $\left(\frac{3}{2}\right)^{\frac{2}{3}} \approx 1.3104$ times larger than that of the associated planet. This suggests that the resonant zones for Venus, Earth, and Mars are centred at 0.9478 AU, 1.3104 AU, and 1.9966 AU, respectively. Assuming a resonant eccentricity of ~0.1, the resonant width (the spread in *a* on either side of the center of resonance in which resonance is to be expected) is approximately $\Delta a/a=7.2\times10^{-4}$ (Murray and Dermott, 1999). These widths should delineate the resonant zones, and we examined behavior of known objects with osculating semimajor axes placing them, within their uncertainty limits, in the respective resonant zones of the inner planets. For all three resonant zones, a number of objects with *a* immediately outside and inside the resonant zone (on the order of the zone widths) were also examined to verify that they were circulating and ensure completeness.

As the 3:2V and 3:2E objects have relatively short lifetimes in resonance due to planetary close encounters, we do not expect the Yarkovsky effect to have noticeable effects on these objects (Connors et al., 2002). The 3:2M objects, however, with their much longer lifetimes are possibly noticeably changed by the Yarkovsky effect. This is discussed further in section 4, where we discuss size and albedo, physical properties that contribute to the Yarkovsky effect.

2.1 Method I

In our search for inner solar system 3:2 objects, we took candidates to be those objects given by Astdys as having their osculating *a* in a resonant zone at epoch 54400 (MJD). As well, we examined a similar number of objects immediately inside and outside the resonant zones for completeness. All candidates were integrated backwards and forwards in time together at a 750 day output interval and a one day time step for at least twenty thousand years, from which it was possible to examine each resonant argument. These integrations were conducted using the Mercury integrator package, and nine planets were included for completeness (Chambers, 1999). Further calculations on 3:2M object 2004 CN_{50} were also done with the Wisdom-Holman algorithm (Wisdom and Holman, 1991) with a time step of seven days (see below). An object is taken to be in 3:2

resonance if its associated resonant argument can be seen to librate about 180º for at least half a resonant period.

Most 3:2 objects discussed here and throughout this paper have well-determined orbits. We take well-determined orbits to be those whose arc length is greater than their orbital period. Orbits of only three 3:2 objects in this paper are not well-determined: 2001 QE_{96}, $2005GP_{21}$ and 2007 JZ20 have observational arcs of 29, 40, and 93 days, respectively. These objects have 1-sigma variations in *a* equal to approximately their respective resonant width, and are therefore relatively unreliable. Most 3:2 objects, and all of the 3:2M objects, have observational arcs over 1000 days. The 3-sigma variation in *a* for these objects is on the order of 10^{-3} of the respective resonant width. We have performed clone studies which would be needed to understand chaotic effects only for 3:2M object 2004 CN_{50}. Since the behaviors described characterize many objects, we do not claim to provide exact descriptions of the distant past or far future of any given object, but that the dynamics described apply generally to the various sorts of inner solar system 3:2 librators.

2.2 3:2Venus resonators

In the case of Venus, the asteroid 5381 Sekhmet has been noted to be in 3:2V (V for Venus) mean motion resonance (Bykova and Galushina, 2002). By coincidence, this object has been found to share (Nolan *et al.*, 2003; Neish *et al.*, 2003) the binary nature of Pluto-Charon, whose recently discovered third and fourth satellites (Weaver et al., 2006) are quite small. Sekhmet's eccentricity of 0.296 is similar to other 3:2V resonators discussed below, but its inclination of 49º is considerably larger than that of any other object studied here.

We have further found that asteroids 2000 ET_{70}, 2005 BO_1, and 2006 SF_6 are in 3:2V resonance. Apart from having *a* close to 0.9478 AU required for resonance, the orbits of these objects are not markedly similar (see Table 1). Sekhmet and 2000 ET_{70} have resonant argument libration periods of approximately 400 years, while the same value for 2005 BO_1 and 2006 SF_6 is closer to 200 years. We find that Sekhmet is the longest lived resonator, staying in resonance for 20,000 years, while 2000 ET_{70} is the shortest as it is nearing the end of its 2000 year resonance. Sekhmet at present makes close approaches to Earth approximately every 12 years and 2000 ET_{70} makes paired close approaches, also every 12 years at present. The effect of these on orbital parameters is shown in Fig. 3. Such approaches limit the lifetime of these objects.

[Figure 3]

Solar system objects with *a* interior to the Earth's (such as 3:2V objects) are difficult to discover due to being often near the Sun in the sky. With large parts of their orbits being inside that of Earth, there is a selection effect acting against discovery of 3:2V objects, especially those with small eccentricities.

2.3 3:2 Earth resonators

We found six known objects to currently be in 3:2E mean motion resonance. For example, asteroid 67367 mentioned above is in resonance with Earth. It was discovered on June 7, 2002 by the LINEAR project (Stokes *et al*., 2000) and originally designated 2002 LY$_{27}$, but its orbit could subsequently be traced back to 1976, and thus is extremely well known. Its orbit in space relative to Earth bears great resemblance to that of Pluto relative to Neptune as may be seen in Fig. 1, and the minimal orbital intersection distance (MOID) is 0.045 AU. However, like Pluto, its longitude of perihelion and position within its orbit keep it much farther than the MOID from its associated planet. The elements, some also subject to libration in the 3:2 resonance, have also been observed to vary due to resonance. The elements e, i, and a of 67367 and their computed variations over 600 years, showing more than one libration, are shown in Fig. 4.

[Figure 4]

Having examined the orbital properties of other objects with similar osculating semimajor axis a, we also find asteroids 2005 GP$_{21}$, 2001 QE$_{96}$, 2000 YJ$_{11}$, 2007 JZ$_{20}$, and 2002 AV$_{31}$ to be librating in 3:2E resonance. As shown in Table 1, these librating objects are currently close to the nominal a value of 1.3104 AU required for resonance. The eccentricities of all but 2001 QE$_{96}$ are in the limited range of 0.212 to 0.250. The inclinations of all except 2007 JZ$_{20}$ (which has an inclination of 40°) range from 7° to 19°, typical values for asteroids. In some ways, the e and i values are reminiscent of those of Pluto, but its libration of $\boldsymbol{\omega}$ about 90° is associated with the Kozai resonance (Nesvorny and Roig, 2000). None of the Earth-resonant objects has values of $\boldsymbol{\omega}$ near this value.

Since asteroids in the 3:2 libration mode spend little time near opposition as seen from Earth, there is an observational selection effect acting against their discovery. This is made clear in Fig. 1 where part of the motion is indicated by dots equally spaced in time. As already pointed out by Cohen and Hubbard (1965), the loops in the corotating frame trajectory are the location relative to the planet where such objects spend a relatively large amount of time, and opposition a considerably lesser amount. As seen from Earth in the case of 3:2E, elongations far from opposition are the best places to search for 3:2E objects, yet most present searches concentrate on the opposition region. Thus the objects mentioned here are likely only representative of a larger population, undersampled by current searches, although we do not estimate the unbiased quantity of objects in this population.

2.4 3:2 Mars resonators

Seventeen asteroids have been found to be in 3:2M resonance with Mars: these are 12008, 37479, 76828, 133039, 141096, 155725, 1999 RO_{37}, 2002 SS_{28}, 2002 GO_6, 2002 TQ_{31}, 2003 EP_{43}, 2003 GK_{21}, 2004 AH, 2004 BS_{58}, 2004 CN_{50}, 2004 DJ_{25}, and 2005 CU_5. Their orbital parameters are listed in Table 1.

There are enough 3:2M resonators that some statistical aspects may be meaningfully examined. The nominal width of libration is 0.0014 AU, corresponding to $\Delta a/a=7.2\times10^{-4}$ for zero inclination and other restrictions (Murray and Dermott, 1999). The limits where objects with similar orbital parameters to those librating no longer librate are *a*=1.9948 AU and *a*=1.9978 AU (in osculating semimajor axis). We therefore find an experimental resonant width of 0.0015 AU about an average *a* of 1.9963 AU, which compare well with the nominal values given above. As can be seen in Table 1, the maximum and minimum eccentricities for 3:2M objects are 0.42408 and 0.09636 with an average of 0.23. The maximum and minimum inclinations are 29.745º and 6.739º with an average of 16.3º. The true test of trapping in the resonance is libration, and we have found that of 27 asteroids in the 3:2M semi-major axis range indicated above, 17 show long-term resonant behaviour. Several other objects show a relatively slow circulation of the resonant argument of the resonance and may be nearly trapped, as these objects loosely switch in or out of resonance over the fourty thousand years studied (but are not included as resonators as they do not currently show 3:2 libration). Over the small resonant range in *a*, the 3:2 resonance has noticeable effects. However, we do not find that there is an excess of known asteroids in the vicinity of the resonance nor is the region of 3:2 trapped bodies prominent on a graph of *a* versus *i* or *e* as is the case for the plutinos (e.g. Luu and Jewitt, 2002).

Despite being located slightly inside (i.e. sunward of) the 4:1J mean motion resonance with Jupiter at 2.06 AU, the position of a Kirkwood gap depleted of asteroids, 3:2M resonator dynamics are dominated by Mars, and likely have been on the time scale of the Solar System. None of the 3:2M objects have $e > 0.54$ necessary to cross the orbit of Earth. Evans and Tabachnik (2002) suggested that there could be (nonresonant) stable zones between Earth and Mars harboring primordial material. We suggest that Mars-zone solar nebula material may be trapped in this resonance, and it would be useful to conduct spectral studies. However, until simulations with timescales on the order of the age of the Solar System are conducted, this remains speculation.

Since these Mars-resonant objects can have favorable oppositions as seen from Earth, their orbits tend to be well-determined and there is no particular selection effect acting against their discovery. Since there are preferred spots in the Mars corotating frame (the “loops”) where 3:2M resonators are slow-moving with respect to Mars, there are places where 3:2M resonators are most likely to be discovered. However, the loop zones are quite large in extent on the sky and directed searches in these areas are not likely to be fruitful compared to current all-sky searches already underway (e.g. Rabinowitz et al., 1998; Stokes et al., 2000).

3. Origin and lifetime

An asteroid is planet crossing if attaining perihelion distance $q = a(1-e)$ or aphelion distance $Q = a(1+e)$ results in crossing at least the immediately inner or exterior planet’s orbit. Due to relative motions of orbits, an asteroid of moderate or low inclination that has perihelion inside the aphelion distance of an inner planet, or aphelion outside the perihelion distance of an exterior planet, will generally interact with that planet relatively quickly. Such interactions can lead to changes in the resonance and presumably, more rarely, larger changes that correspond to injection or extraction into a resonance.

In this paper, libration amplitude for 3:2 resonance is taken as the difference between the maximum and minimum resonant argument values for four thousand years about the present. If an object in 3:2 resonance has sufficient libration amplitude, interactions with the resonant planet (e.g. Mars in 3:2M) can also lead to changes in the resonance and ejection. As well, a large libration amplitude means that the object is already close to leaving resonance, and a close approach with any planet then becomes more likely to eject the object from resonance. We find that low libration amplitude is correlated with long resonance lifetime for all objects, although the three different populations studied here are in general not studied long enough to provide a full description of the relationship between lifetime and libration amplitude. Libration amplitudes are given for each object in Table 1.

3.1 Method II

To study changes in orbital behavior due to close encounters (e.g. changes in libration amplitude, ejection from 3:2 resonance, etc.), the Mercury integrator was used with large output time steps (50 days) until near the time that a change of interest was noted. Small output steps (1 day) were then used for approximately 2000 years on either side of the encounter to study the circumstances. Such small steps are needed if close approaches in the inner solar system are to be properly characterized.

Data given by the above Mercury integrations for twenty thousand years either side of the present (discussed in section 2) was used to calculate libration amplitudes and resonance lifetimes. This was accomplished using an automated program to calculate the amplitude, and a visual inspection of the resonant argument to determine if and when the object entered and left resonance in the given time frame. Periods of uninterrupted resonant libration are used to estimate lifetimes.

For Mars resonators, objects were also integrated using the Mercury integrator package back in time one million years at a timestep of two days and an output of 3000 days. Those objects with resonance on the order of one million years were taken to be long-lived. As opposed to the resonant lifetimes mentioned directly above, lifetimes over the scale of a million years are not required to be entirely uninterrupted; if the object leaves resonance for less than a few libration periods and then returns to libration, we take this as maintaining resonance.

To begin to examine the long-term behaviour of 3:2M objects, the 3:2M resonator with the smallest resonant amplitude 2004 CN_{50} (48°) was selected for further examination. A suite of 20 clones of 2004 CN_{50} was chosen randomly within the orbital error limits as given by the covariance matrix for this body from the NeoDys website (cited above) on Sep. 30, 2007. These clones represent possible orbital trajectories for 2004 CN_{50} that deviate from the nominal orbit only by amounts within the current error bounds on the orbit. This suite thus provides some insight into the possible evolution of 2004 CN_{50} once one accounts for the fact that the orbit is known to only finite accuracy.

This suite of clones was integrated backwards for 12 million years with a symplectic integration algorithm based on the Wisdom-Holman method (Wisdom and Holman 1991), with close approaches handled by the hybrid method (Chambers 1999). The integration used a step size of seven days in a solar system that included eight planets, all mutually interacting gravitationally, though the effect of the clones on the planets was neglected.

We now proceed to discuss the behaviours found, and how they differ, planet by planet.

3.2 3:2 Venus resonators and Earth

By librating in *a* about 0.947 AU, 3:2V need only a very small eccentricity of about 0.05 to become Earth crossing. In addition, Venus is close to an 8:5E mean motion resonance with Earth (Chapman, 1986), so that any object resonant with Venus is already nearly in resonance with Earth. 3:2V librators are therefore presumably injected or extracted mainly by Earth encounters, and the 3:2V objects discussed in this paper have close encounters with Earth regularly, and short lifetimes in resonance. It is possible that future objects will be found which are relatively long-lived in the resonance, but at present it does not merit much discussion. The nearly 49º inclination of 5381 Sekhmet can imply an origin through a very close approach and strong scattering at Earth. Whether its binary nature arose in this scattering or is a constraint speaking against the scattering hypothesis is impossible to say at this time. In Fig. 3, the effects of Earth encounters on the elements of the lower inclination object 2000 ET_{70} are shown near the present when it is in resonance. Over a forty thousand year time scale studied, this object enters and leaves resonance intermittently, after first entering it in approximately 5000 BC as shown in Fig. 5. At this time, effects of a series of close encounters accumulated stochastically to increase *a* into the resonant region. Although our integrations do indicate one very close approach to Earth at approximately 200 AD, Sekhmet similarly appears to have been injected by repeated close approaches to Earth, which decreased *a* into the resonant region (not shown). Considering the cases of Sekhmet and 2000 ET_{70}, it appears that the cumulative effect of close approaches can play a role in injection or extraction in 3:2V resonance. None of the objects had close approaches to Mercury.

[Figure 5]

None of the 3:2V objects stayed in strict resonance for the length of the 40000 year integrations. Sekhmet is the longest lived of these objects at 26000 years in resonance. The libration amplitude of this object at present day is 68º. The lifetimes of 2005 BO_1, 2006 SF_6, and 2000 ET_{70} are 13000, 10000, and 2000 years, respectively; and the amplitudes of these objects are 137º, 154º, and 160º, respectively. An inverse correlation of lifetime and amplitude of libration is seen for all objects, although there are not sufficient objects to look at statistics meaningfully.

The smallest of the 3:2V objects is given by EARN to be 100m (see Section 4 below). Using an eccentricity of 0.3 and assuming a rotation period of approximately five hours, Spitale and Greenberg (2000) calculate that *a* should not change by more than 0.5 km a year as a result of the Yarkovsky effect. At this rate, it would take approximately

10^5 years for *a* to change an amount on the order of the 3:2V resonance width. As can be seen in Fig. 5, frequent close approaches limit the lifetime of 3:2V resonance, and one would not expect the Yarkovsky effect to be noticeable on these time scales. However, for smaller undetected objects on the order of 1m and 10m, it is possible that the Yarkovsky effect could be significant.

3.3 3:2 Earth resonators and Mars

3:2E resonators could in principle be injected or ejected by Mars or Venus encounters, and could be ejected by Mars. With sufficient amplitude libration so as to lessen the resonant avoidance mechanism, 3:2E objects could also be scattered by Earth itself, although we did not find any examples of this. The perihelion of Mars is approximately at 1.38 AU, while the aphelion of Venus is at 0.728 AU. Since in this case $a \approx 1.3104$, Venus crossing requires $e \geq 0.45$, while Mars crossing requires only $e \geq 0.05$ approximately. Of the six known 3:2E resonators, only 2001 QE_{96} has an eccentricity (0.028) less than the value to cross Mars, while none cross Venus. Thus Mars should play a role in the orbital evolution and presumably injection or extraction of most of the known objects.

For Earth 3:2 resonators, close approaches to the immediately exterior planet (Mars) are thus the determining factor in stability. The near-term behavior of 2000 YJ_{11} is shown in Fig. 6. This object has a good orbit, with many observations since its discovery, so the near-term behavior shown should be close to reality. When the geometry is appropriate, close encounters with Mars take place and change the resonant argument of the 3:2E resonance, most notably in amplitude. On the timescale shown, some very close approaches to Mars, less than .01 AU, take place and step-like element changes (for example visible in *a* in Fig. 6) take place. Minimum distance to Earth also decreases at these times, although never gets closer than 0.1 AU. Over this time, there were no close approaches to Venus, and Mars is the only perturbing body with a large effect on the object. As Mars is a less massive perturbing body than Earth, and as the 3:2E objects have a longer period than the 3:2V, one would expect the lifetime of 3:2E inner solar system resonators to be greater than the 3:2V. Although we do not find any clear examples of injection into 3:2E by Mars, close approaches with Mars are the deciding factor for 3:2E resonant lifetime and are most likely the main injection and ejection method. The resonant argument traces for all objects examined for 3:2E resonance can be seen in Fig.7.

[Figure 6,7]

Only one of the 3:2E objects (2002 AV_{31}) stayed in strict resonance over the 40000 year integrations. Its current libration amplitude is 172º. If this object is taken as an outlier, objects 67367, 2007 JZ_{20}, 2001 YJ_{11}, 2001 QE_{96}, and 2005 GP_{21} show an anti-correlation between their libration amplitudes and lifetimes. As only 2005 GP_{21} entered and left resonance on the time frame of the integrations (lasting 12000 years and currently librating with an amplitude of 350º), the values for the other objects' lifetimes are bounded by an exit or entry and the integration limits. They accordingly have minimum lifetimes of 31000, 27000, 25000, and 18000 years, respectively. The libration amplitudes are133º, 174º, 175º, and 232º, respectively.

For similar reasons to those given in Section 3.1, the Yarkovsky effect is not likely to have a significant effect on the evolution of known 3:2E objects, given that the smallest known 3:2E object is 100m in diameter (Spitale and Greenberg, 2000). The lifetime of 3:2E objects is dominated by close interactions with Mars.

3.4 Stability of 3:2M resonators

For the integrations covering 40000 years, 13 of the 17 3:2M objects maintained resonance over the entire period. All of these 13 objects have libration amplitudes below 150º, and six of them have amplitudes below 100º. The lowest amplitude object is 2004 CN_{50}, with an amplitude of 48º. All objects which left resonance at least once during the time span currently have amplitudes above 160º.

When integrated backwards one million years, all except two of the 3:2M objects (12008 and 2004 AH) were shown to be stable for at least the last that time. We have discussed the likely injection into and extraction from resonance for the case of Venus and Earth 3:2 resonant objects and infer that the presence of the next exterior planet is involved. In the case of Mars, there is no large body in a suitable position for a similar mechanism to operate. The 3:2M resonators in this paper have orbits crossing that of no planet but Mars, and 3:2M resonators generally avoid Mars close encounters due to the resonance dynamics.

We proceed to discuss a few unstable cases, to discuss the more stable situations, and finally to begin to examine whether stability on a timescale similar to the age of the Solar System is likely.

3.4.1 Unstable or nonresonant objects near 3:2 Mars resonance

3:2M resonators are not subject to planet crossing in general. Crossing Earth would require $e \approx 0.54$, and most of the 3:2M resonators have low eccentricity. Objects not in resonance and with sufficiently large eccentricities may be discerned in Fig. 8, as those in the resonant a range that have rapidly circulating resonant arguments. We have not numbered these nonresonant objects, nor included them in Table 1. However, these high eccentricity cases merit some discussion.

[Figure 8]

The object 2000 PD$_3$ is located in semimajor axis below the resonant objects 141096 (25 in Table 1 and Fig. 8). It has $e \approx 0.59$ and $i \approx 8^{\circ}$, and it has frequent close encounters with Earth. Although at the present time its resonant argument is changing slowly near 180º, close encounters with Earth prevent it from falling into resonance. Somewhat similar behavior is shown by 2004 EC, with slightly higher a than 1999 RO$_{37}$ (23 in table and Fig. 8). This object has $e \approx 0.86$, making it Earth-crossing and Venus-crossing. An inclination of about 35º keeps it out of the plane of the planets for large amounts of time, but it still has frequent close encounters with Earth. Lower in the range of a which could permit resonance, 86819 is two below 2003 GK$_{21}$ (15 in table and Fig. 8) and has $e \approx 0.51$, and can approach Earth closely, explaining its instability. Those nonresonant objects in the resonant range which are not mentioned here do not have high eccentricity and circulate relatively slowly, and we do not offer an explanation for why they are not currently resonant.

3.4.2 Characteristics of objects in 3:2 Mars resonance

2004 CN_{50} (object 18) was noted as having been in the 3:2M resonance for one million years. This object has very low libration amplitude, meaning that it does not deviate much from the resonant condition of Mars avoidance. In contrast, 2002 TQ_{31} (object 17) has a very large libration amplitude and at approximately 4000 years in the future slips out of resonance. Being in the resonance does not greatly enhance its stability as it permanently leaves resonance approximately 16,000 years in the future, and it may be considered to be on the 'edge' of 3:2M resonance. Asteroid 2001 TN_{103} shows slow and relatively stable circulation just below 2003 GK_{21} (object 15 in Fig. 8) and has eccentricity of 0.08. Other slowly circulating objects, such as 2000 SH_{47} just above 2003 EP_{43} (object 14 in Fig. 8), have relatively low eccentricities and appear stable. These objects have slow circulation of the resonant argument which may switch to large-amplitude libration. In contrast, the 3:2M resonators are usually objects with a larger eccentricity. Exemplified by 2004 CN_{50} with $e \approx 0.18$, they can be stable in the resonance if their libration amplitude about 180° is on the order of 90° or less, making them have Pluto-like orbits with avoidance of the associated planet. Since Mars' orbit itself has large eccentricity, the limits on the amplitude of the angular argument before disruptive encounters with the planet are more severe than in the case of most other planets. This is reminiscent of the effects of Mars' eccentricity on its own co-orbital objects (Connors et al., 2005).

3.4.3 Long-term stability of objects in 3:2 Mars resonance

The question of whether or not material in the 3:2 resonance with Mars is stable on billion year time scales is of interest. If such stability exists, then bodies trapped in this resonance could be primordial, having been in it since the formation of the planets. This would make them possibly the only primordial small bodies still extant closer to the Sun than the asteroid belt (with the possible exception of Phobos and Deimos) and thus potentially holding important chemical and dynamical information about the early Solar System.

As described in Method II, we integrated 2005 CN_{50} and 20 of its clones backwards for 12 million years. Through inspection of the resonant argument it was seen that only 10 of the 20 clones had remained in the 3:2 resonance for the full length, indicating that we cannot conclusively deduce that 2004 CN_{50} has been in the 3:2 resonance over 10^7 year time scales. Many of the other clones show relatively stable resonant amplitudes during the integration. It may be that as our knowledge of the actual orbit of 2004 CN_{50} (or other 3:2 resonators) improves, they will be found to reside in stable niches of the phase space. As 2004 CN_{50} can currently be considered the most stable 3:2M object due to its lowest 3:2 libration amplitude, the primordial nature of most

3:2M objects looks doubtful. However, integrations over longer periods with a larger suite of clones are needed to determine the possibilities of 3:2M primordial objects.

Extrapolating Spitale and Greenberg's (2000) results for an object with a diameter on the order of a kilometre, as is the case for 3:2M objects, the Yarkovsky effect has possible noticeable effects over the time scale considered here. For an object with a diameter of one kilometre and eccentricity of 0.3, one would expect a to change about 0.02 km per year, and would thus change by the 3:2M resonant width in approximately 10^7 years. The Yarkovsky effect can play a significant role in moving nonresonant objects, although its ability to move an object already bound in a specific resonance is not clear. For this reason, we do not include the Yarkovsky effect in our simulations, and only note its importance for large timescales. The Yarkovsky effect is much more likely to be of importance for smaller as-yet-unobserved 3:2M objects.

The identification of the 3:2M resonators as a class of objects allowing study of the 3:2 mean motion resonance relatively free of influence from other planets is interesting in its own right. Their potential stability over periods of time long enough to allow them to trap material from the early Solar System's terrestrial planet formation zone could also be significant.

4. Physical Properties

In general, the physical properties of the objects identified here have not been thoroughly investigated. For completeness we discuss the objects for which we have found spectral and taxonomic information, and any implications that follow from this. Where possible, information has been attained from EARN.

3:2V object 2000 ET_{70} has been classified to be of X type (Whiteley, 2004). Based on its H magnitude and a typical asteroidal albedo, it would be about 1 km in diameter. Radar observations of binary 5381 Sekhmet (Nolan *et al.*, 2003; Neish *et al.*, 2003) permitted not only to determine a very good orbit but also to show that the primary is about 1 km in diameter and the secondary about 300 m, although the binary orbital period was not clearly determined. Later spectral observations suggest a V type classification and are consistent with the radar-derived sizes given a reasonable albedo (Davies et al., 2007). Neither X nor V classification is unusual among near-Earth asteroids, and absence of V-types in Mars-crossing orbits has been taken as diagnostic of rapid injection of this class to the inner Solar System by resonances (Binzel et al., 2004). Given the small number of objects, and the likelihood of orbits evolved since injection, the Tisserand parameter approach of those authors in determining source regions is not helpful. That the presence of X-type may indicate a cometary source cannot be excluded. Based on observed magnitudes and assumed typical asteroidal albedos, 3:2V object sizes as given by EARN range from about 1.5 km diameter for Sekhmet to 100m for 2005 BO_1.

No 3:2E resonators appear to have intercalibrated color measurements available. Based on observed magnitudes and assumed typical asteroidal albedos, their sizes range from about 1 km diameter for 67367 down to about 100 m for 2001 QE_{96}.

Among 3:2M resonators, no suitable color measurements could be found, although in the case of the largest member, 12008 Kandrup with H=12.8, these would be easy to obtain. This magnitude implies a diameter of roughly 10 km. The other known 3:2M resonators are smaller, with a maximum H=17.9 for 133039 and minimum H=15.9 for 76828, not counting outliers Kandrup and 2004 AH (H=19.1). This makes them kilometer-scale objects, with little spread and no evidence for a power-law distribution. Asteroid 37497 was discovered in 1971; 12008 Kandrup was officially discovered in 1996, but with earlier detections found as far back as 1949; the remainder of this class of asteroid has only been found since 1999 and has an average magnitude near the current limit of large-area search programs. This likely explains the lack of a size distribution since only the largest objects are currently known.

With this paucity of information, it is useful to indicate what would constitute useful observations. In the case of Venus and 3:2E resonators, scattering mechanisms have most likely been active and as more objects and color data become available, one would expect a distribution of objects much like the NEOs discussed by Binzel et al. (2004). We hypothesize larger residence times in the 3:2M resonator region, and speculate that these would show less mixing and an inner belt composition.

5. Other behaviors

For some objects similar to 3:2 resonators, we have noted libration of the resonant argument about 0º rather than 180º at certain times for six objects. These objects are 2006 TS_7 and 2001 SQ_{263} in the Venus resonance range, and 1996 AJ_1, 2007 JZ_{20}, 1999 VF_{22} and 2001 HB in the Earth resonance range. None were found in the Mars resonance range during the period studied. At the times of the libration about 0º, these objects have high eccentricities of 0.58 and 0.48, and 0.50, 0.70, and 0.79, respectively. Only one of these objects also displays 3:2 resonance (2007 JZ_{20}), and we do not include the others as resonators, as being "plutino-like" requires libration around 180º for resonant protection from encounters. These objects have arcs of 18, 20, 3283, 126, 23, and 3290 days respectively, so only 2001 HB and 1996 AJ_1 have well established orbits. Such objects are usually discovered on close approaches to Earth, so only a small portion of the space available to them has been sampled. Thus they could be representative of a larger population.

The highly eccentric objects open the possibility that relatively stable 3:2 resonant motion can persist at high eccentricity since encounters with the associated planet can be always distant. For example, 1996 AJ_1 has very close approaches in this epoch with Mercury, Mars and Venus, but not very close to associated planet Earth. Such potentially stable, high-eccentricity orbits are disrupted in our solar system since they imply multiple planet crossing. This may not be the case in certain exoplanetary systems, so that long-lived high eccentricity 3:2 resonance of this sort may be found.

6. Conclusions

The outer Solar System 3:2 librators were likely captured into 3:2N resonance due to resonance sweeping accompanying radial migration of Neptune (Malhotra, 1995). In some ways this mechanism is similar to that proposed for the origin of Mercury's spin:orbit coupling, in which changing orbital eccentricity sweeps through the conditions needed for resonant lock (Correia and Laskar, 2004). Our understanding of resonance sweeping continues to improve, but migration seems the most likely way in which to understand the present-day dynamics of Pluto and the plutinos (Morbidelli, 2004; Wiegert, 2003). This mechanism is not likely to have been important in the early inner Solar System, and other mechanisms like close encounters are responsible for the dynamics of inner solar system 3:2 librators.

These close approaches with the immediately exterior planet that limit the lifetime of Venus and 3:2E resonators presumably also account for their injection into the resonance. These are likely not primordial objects since the conditions of resonance appear to need a relatively high eccentricity and thus for Venus and Earth, the 3:2 librators are necessarily planet-crossing and subject to disruptive perturbations from Earth and Mars, respectively. On the other hand, some 3:2M resonators appear to have been in resonance with Mars for lengthy periods. If they have been trapped in this resonance since the beginning of the solar system, there could be interesting cosmogonic information associated with them. However, the Yarkovsky effect likely limits the lifetime of small 3:2M objects. A spectroscopic investigation could reveal whether there are any common physical properties possibly reflecting a common formation zone in the solar nebula.

Inner solar system 3:2 librators also offer the possibility to study the dynamics of the 3:2 mean motion resonance with observational timescales allowing libration and other potential subtle details to be measured and compared to theory. Their lifetimes and injection/extraction rates should inform studies of transport of asteroids in the inner solar system. We have also identified somewhat stable 3:2 resonant behavior with libration around zero degrees, possible at high eccentricity. Such motion may merit further study.

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Table 1. Osculating orbital elements on MJD 54400 for four 3:2V objects, six 3:2E objects, and seventeen 3:2M objects, sorted by osculating a. Ω is the longitude of the ascending node, ω the argument of perihelion, and M the mean longitude. The number (#) column corresponds to numbering in Fig. 7 and 8. Elements cited from http://hamilton.dm.unipi.it/cgi-bin/astdys/astibo.

Object	#	a (AU)	e	i (deg)	Ω (deg)	ω (deg)	M (deg)	Libration Ampl. (deg)
2000 ET$_{70}$	1	0.94695	0.123515	22.322	331.201	46.366	237.379	160
(5381)Sekhmet	3	0.94746	0.296148	48.972	58.561	37.419	53.102	68
2005 BO$_{1}$	2	0.94879	0.356033	10.675	113.409	174.115	197.904	137
2006 SF$_{6}$	4	0.94936	0.280439	5.865	228.164	305.51	256.8	154
2005 GP$_{21}$	5	1.30833	0.224393	18.795	10.048	1.303	10.048	350
67367	6	1.30859	0.212823	9.024	264.585	184.719	126.616	133
2001 QE$_{96}$	7	1.31048	0.027852	7.256	150.301	278.972	305.665	232
2002 AV$_{31}$	8	1.31122	0.249831	14.978	119.384	267.25	12.482	172
2007 JZ$_{20}$	9	1.31149	0.335455	40.472	200.537	138.991	45.807	174
2000 YJ$_{11}$	10	1.31216	0.231495	7.264	65.041	339.016	225.539	175
2004 BS$_{58}$	11	1.99479	0.270251	20.86	135.782	252.151	197.938	164
133039	12	1.99506	0.209599	3.541	171.442	292.22	248.086	132
2004 AH	13	1.99507	0.424083	15.989	109.759	300.647	140.295	127
2003 EP$_{43}$	14	1.99518	0.123733	24.329	183.974	309.624	252.898	168
2003 GK$_{21}$	15	1.99597	0.27025	6.741	66.227	345.345	347.67	139
2002 GO$_{6}$	16	1.99599	0.120152	20.384	212.992	130.574	222.41	61
2002 TQ$_{31}$	17	1.99602	0.12082	22.287	197.333	195.982	277.683	169
2004 CN$_{50}$	18	1.99632	0.180733	17.144	252.804	248.573	46.19	48
12008	19	1.99632	0.316425	29.745	263.399	344.836	75.033	111
2002 SS$_{28}$	20	1.99659	0.237259	20.53	252.804	120.241	284.429	146
2004 DJ$_{25}$	21	1.99672	0.268478	6.739	184.842	251.513	166.108	49
155725	22	1.99682	0.344745	17.943	180.347	234.172	159.194	87
1999 RO$_{37}$	23	1.99694	0.313999	20.041	144.369	269.172	283.011	65
2005 CU$_{5}$	24	1.99714	0.09636	20.13	328.621	236.706	291.611	99
141096	25	1.99773	0.104804	12.317	325.284	220.21	315.882	141
37479	26	1.99776	0.278054	7.748	196.606	285.324	21.743	119
76828	27	1.99776	0.221736	11.694	245.389	27.95	258.634	148

Table 2. Osculating orbital elements on MJD 54400 for objects librating around 0º discussed in section 5, sorted by osculating *a*. Quantities given are the same as those in Table 1, except that libration amplitude is omitted. Objects c, 9, d, and e can be seen in Fig. 7.

Object	#	a (AU)	e	i (deg)	Ω (deg)	ω (deg)	M (deg)
2006 TS$_7$	a	0.946647	0.579807	5.465	299.728	225.452	317.866
2001 SQ$_{263}$	b	0.948069	0.491471	3.951	262.368	327.299	292.184
1996 AJ$_1$	c	1.31002	0.781418	2.539	238.147	91.085	5.401
2007 JZ$_{20}$	9	1.31149	0.335455	40.472	200.537	138.991	45.807
1999 VF$_{22}$	d	1.31209	0.73821	3.902	271.397	3.75	144.32
2001 HB	e	1.31386	0.694003	9.293	237.719	196.05	162.851

Figure Captions

Figure 1 Pluto and 67367's orbits in the frame corotating with Neptune and Earth, respectively. The double-loop figure is typical of the 3:2 mean motion resonance. The orbits are not closed due to librational motion over the period of the orbit. The top view in each panel is from the ecliptic north pole. All data created with the Mercury integrator (Chambers, 1999). (a) Two sidereal orbits are shown for Pluto, corresponding to three sidereal orbits of Neptune around the Sun. The time scale is 1510 to 2007 AD. A scale bar of length 30 AU is shown starting at the Sun (centre) and Neptune is shown as a dot 30 AU below the Sun, very close to the average position it holds in its very circular orbit as viewed in this frame. The observed portion of Pluto's orbit is presented as one dot for every three Earth years to illustrate how little of Pluto's orbit has been observed (near leftmost loop). The bottom view is inward toward the Sun looking past Neptune and the positions of Neptune and the Sun are shown as a short bar. The sizes of Neptune and the Sun are not to scale with the orbits. (b) Orbit of asteroid 67367 in the frame corotating with Earth. A period of 3000 days from early 2004 to early 2012 is shown. The Earth is shown as a dot 1 AU below the Sun. The observational record stretches back to 1976, almost four times longer than the period shown. In this short time, the libration is clearly visible. In addition, the final three year (one cycle) part is indicated by dots every 26 days in the top view. These dots make clear the stationary points in the loops. The libration of the orbit is currently counterclockwise relative to Earth.

Figure 2 Comparison of resonant arguments of Pluto and 67367. Note the top and bottom timescales, each in years. Panel (a) shows the 3:2 resonant argument of Pluto from 1500 AD to 30000 AD. The argument librates about 180º, which permits avoidance of near-perihelion close approaches to Neptune. Panel (b) shows the resonant argument of the 3:2E resonance for asteroid 67367, showing libration around 180º with a period of 157000 days (430 years), from 1600 to 2200. Data from Astdys September 2007 .

Figure 3 Elements of asteroid 2000 ET_{70} from 1600 to 2200. From top: semimajor axis *a* (AU) librates around the average 0.948 AU value of the 3:2V resonance; eccentricity *e* and inclination *i* do not appear to be librating over this timescale. Notable features of all parameters are abrupt changes, readily visible in panels with appropriate scale, upon (often paired) close approaches to Earth which repeat each 12 years. These periodic close approaches in general serve to destabilize 3:2V objects.

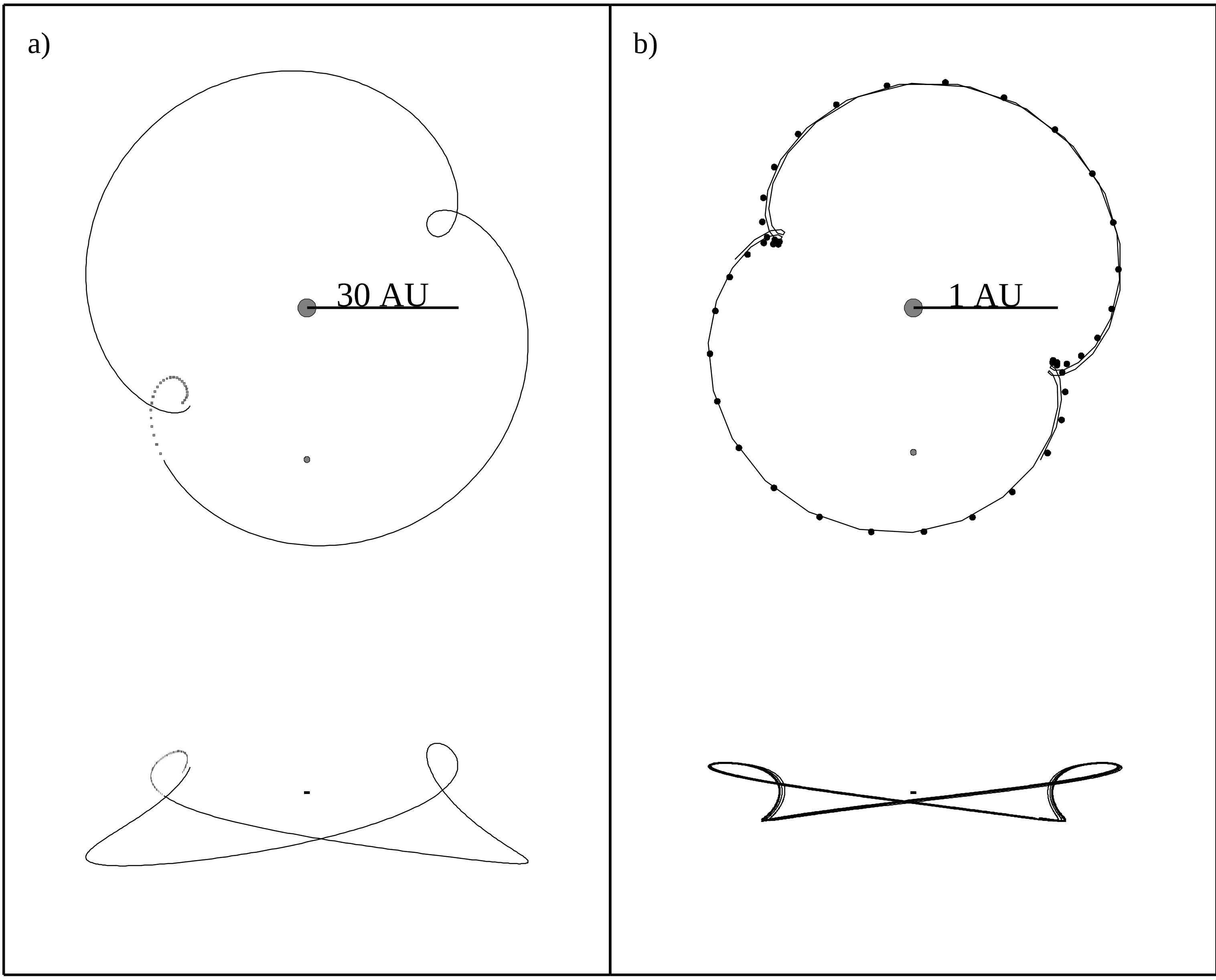
Figure 4 Elements of asteroid 67367 from 1600 to 2200. From the bottom are semimajor axis *a* (AU), which librates around the average 1.3104 AU value typifying the resonance. Eccentricity *e* librates around 0.214 inclination, and inclination *i* (degrees) librates with low amplitude around 9.02º. There are no periodic planetary close encounters for 3:2E objects as there are for 3:2V objects.

Figure 5 Libration of asteroid 2000 ET_{70} near the time of entering 3:2V resonance related to planetary distances. From the bottom are semimajor axis *a*, 3:2 resonant argument, and distance to Earth. There were no approaches to Venus closer than 0.1 AU over this period. At approximately 5000 BC, a series of close approaches to Earth bring 2000 ET70's semi-major axis into the 3:2V resonant range. Further close approaches at approximately 3800 BC destabilize the resonance. Not shown, more close approaches to Earth bring this object back into 3:2V resonance it displays today.

Figure 6 Libration of asteroid 2000 YJ_{11} showing near-term changes in 3:2E resonant behavior related to close approaches with Mars. From the bottom are semimajor axis *a*, 3:2 resonant argument, distance to Earth and to Mars. Step-like changes in *a* at 5300 AD, 5700 AD and 6350 AD are directly related to very close approaches to Mars at those times, evident in the upper panel. Changes in the amplitude of the resonant argument can also be seen to be related to these close approaches.

Figure 7 Resonant argument of 3:2E resonators ordered by semimajor axis. The *a* range of the graphed objects is presented on the vertical axis, and each object's current osculating *a* can roughly be determined from the plot. See Table 1 for exact values. Each subplot is centered about 180º, and the wave-like patterns indicate libration, typical in the central part of the graph (i.e. semimajor axis range). Oblique lines indicate circulation, i.e. objects not in the 3:2E resonance. Sorting was done by osculating elements on MJD 53800 and numbering and lettering are given in Tables 1 and 2, respectively.

Figure 8 Resonant argument of 3:2M resonators ordered by semimajor axis, as in Fig. 7. Those objects in the central *a* range which circulate rapidly or alternate between libration and circulation all have large *e* and thus are affected by other planets (mainly Earth). Sorting was done by osculating elements on MJD 53800 and numbering is given in Table 1.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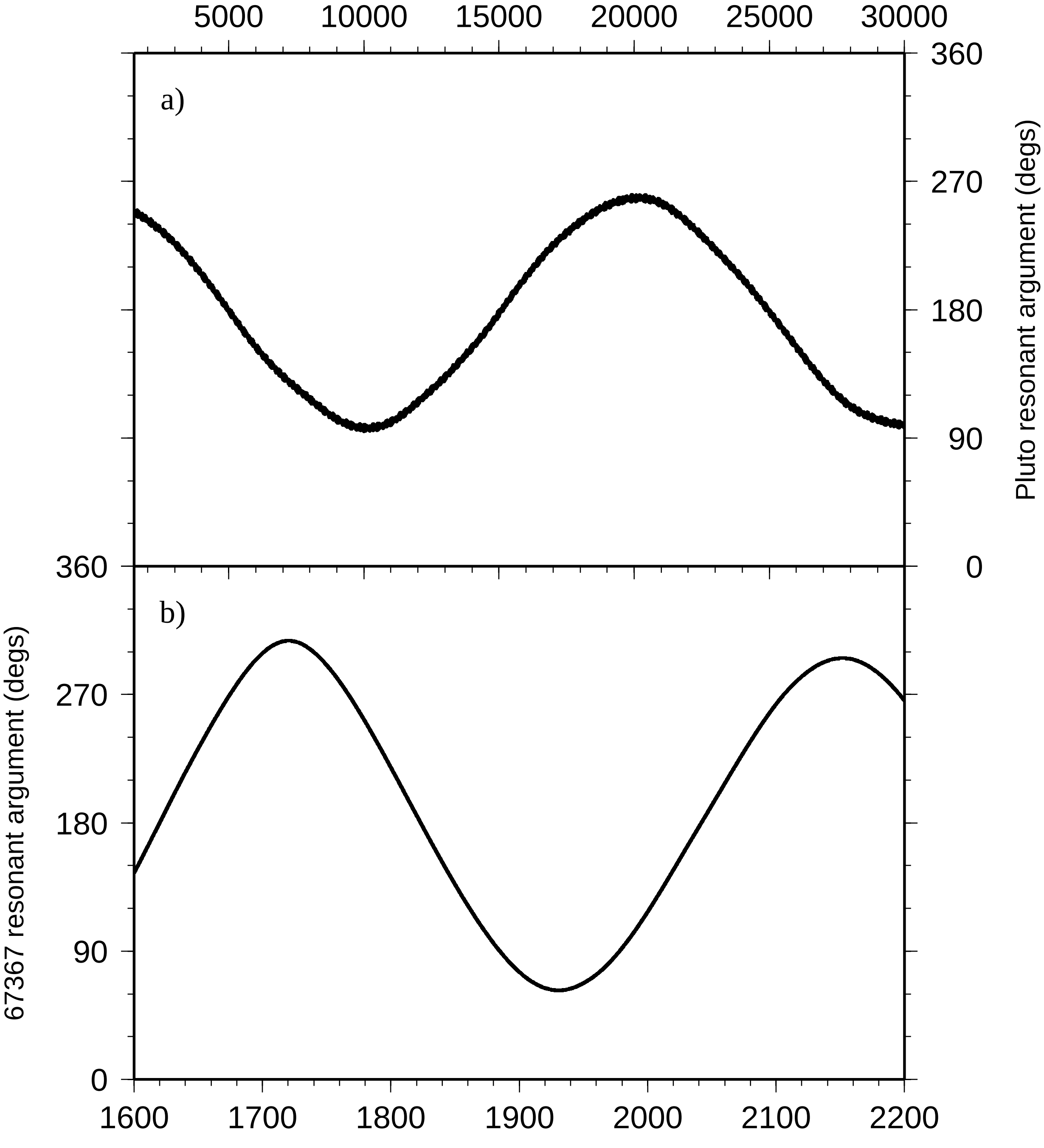
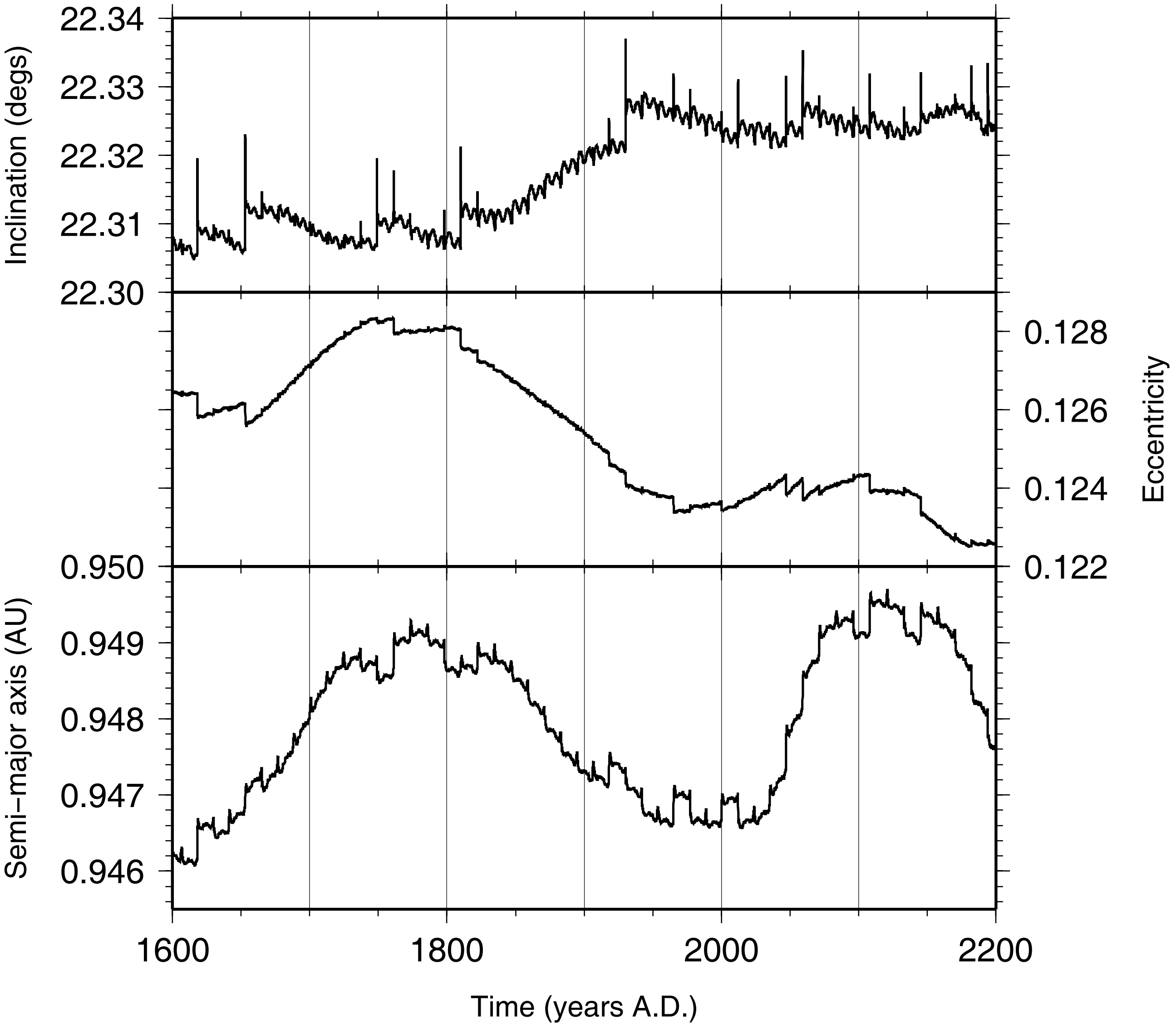
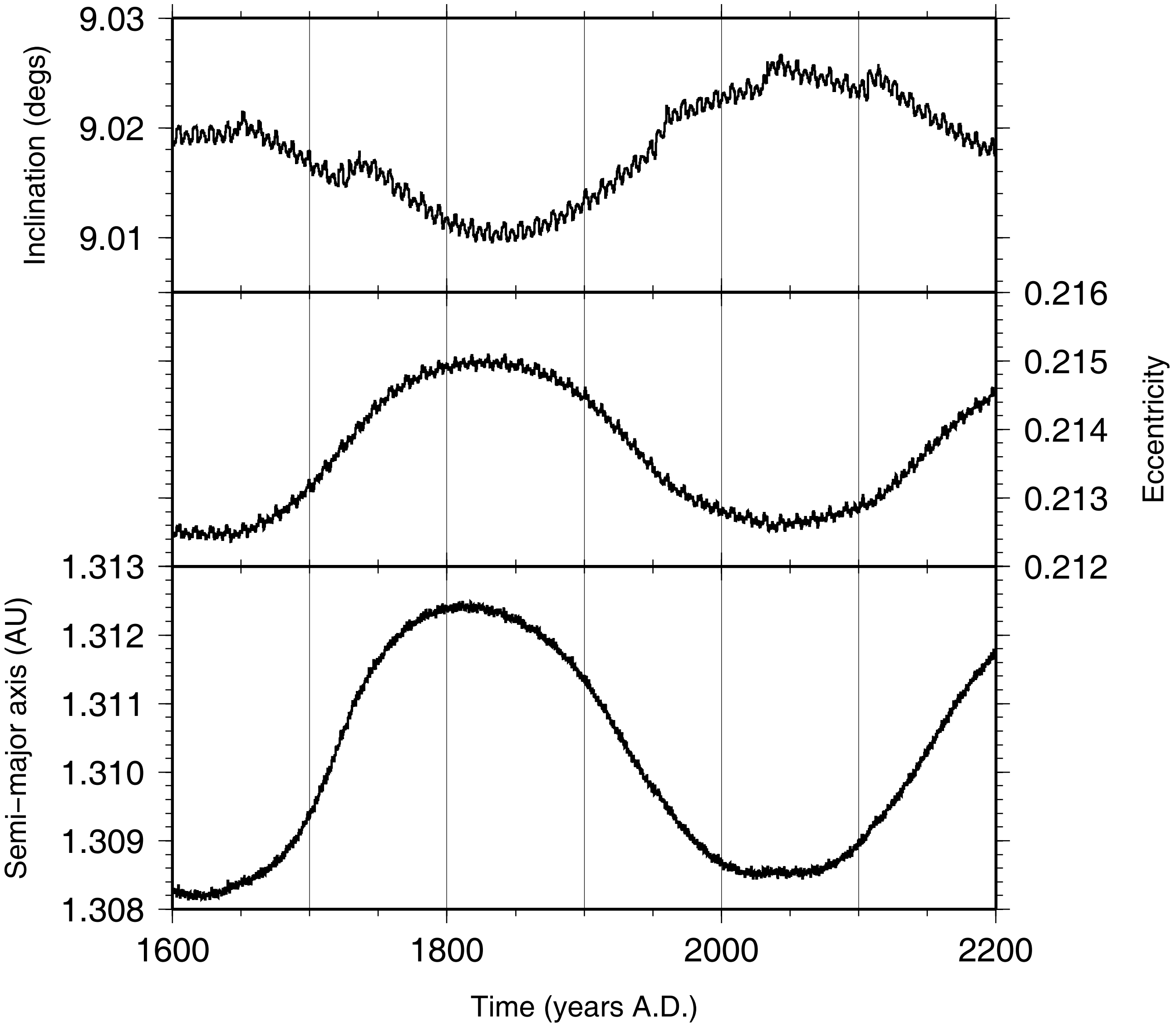
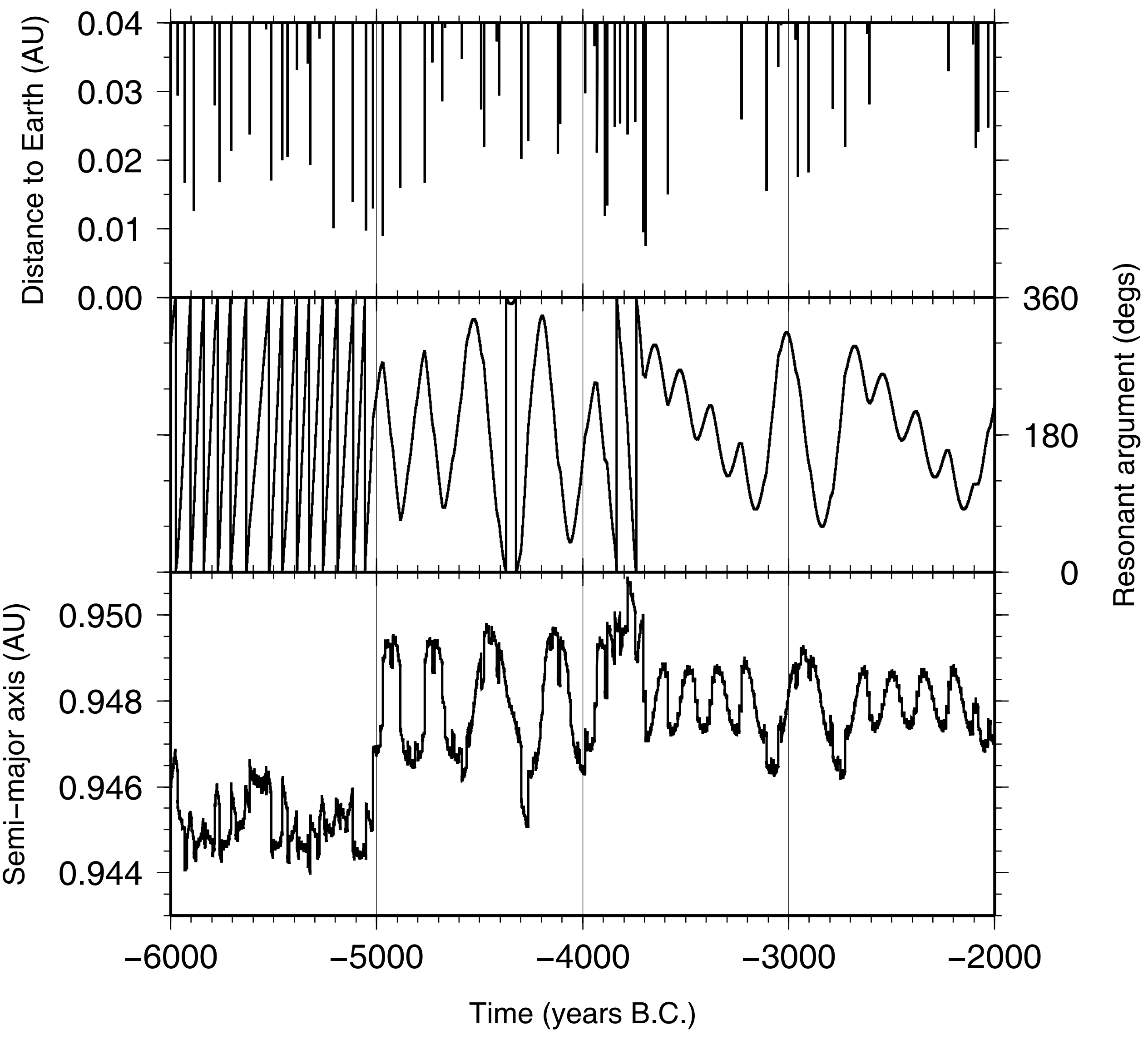
30 AU
b)
1 AU

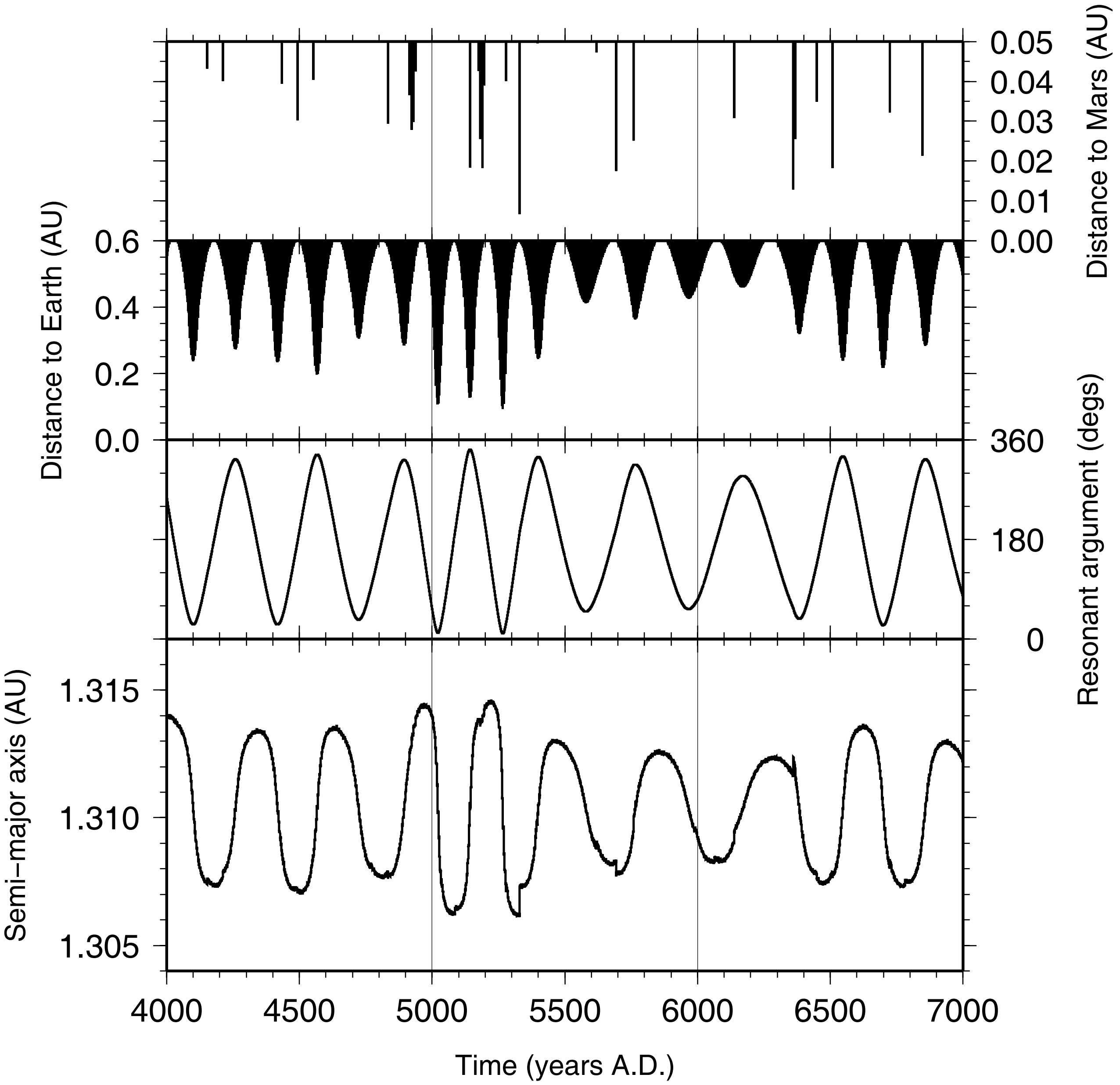

5000
10000
15000
20000
25000
30000
a)
360
270
180
90
0
Pluto resonant argument (degs)
b)
360
270
180
90
0
67367 resonant argument (degs)
1600
1700
1800
1900
2000
2100
2200
Time (years A.D.)


22.34
22.33
22.32
22.31
22.30
Inclination (degs)
0.128
0.126
0.124
0.122
Eccentricity
0.950
0.949
0.948
0.947
0.946
Semi−major axis (AU)
1600
1800
2000
2200
Time (years A.D.)

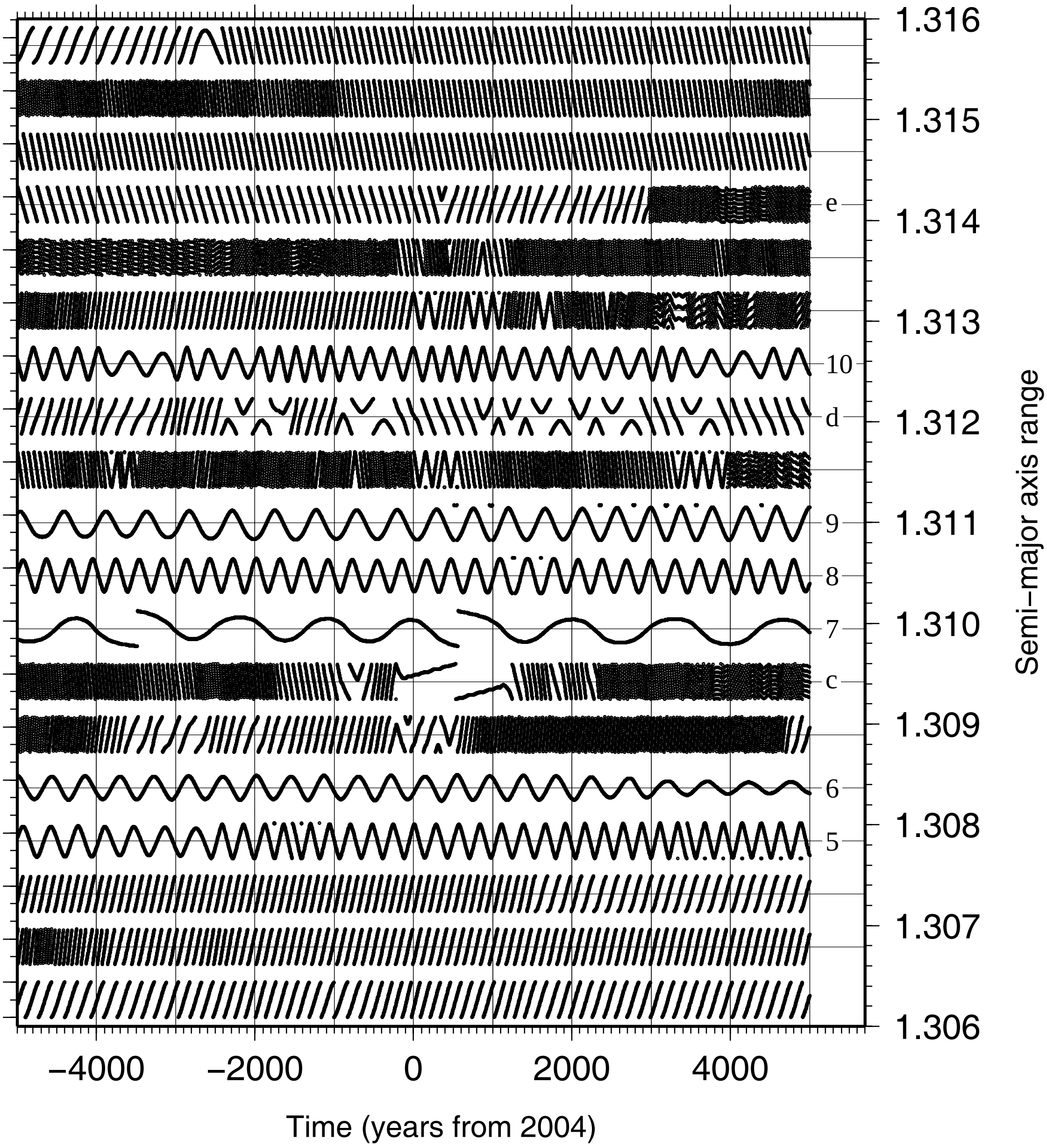


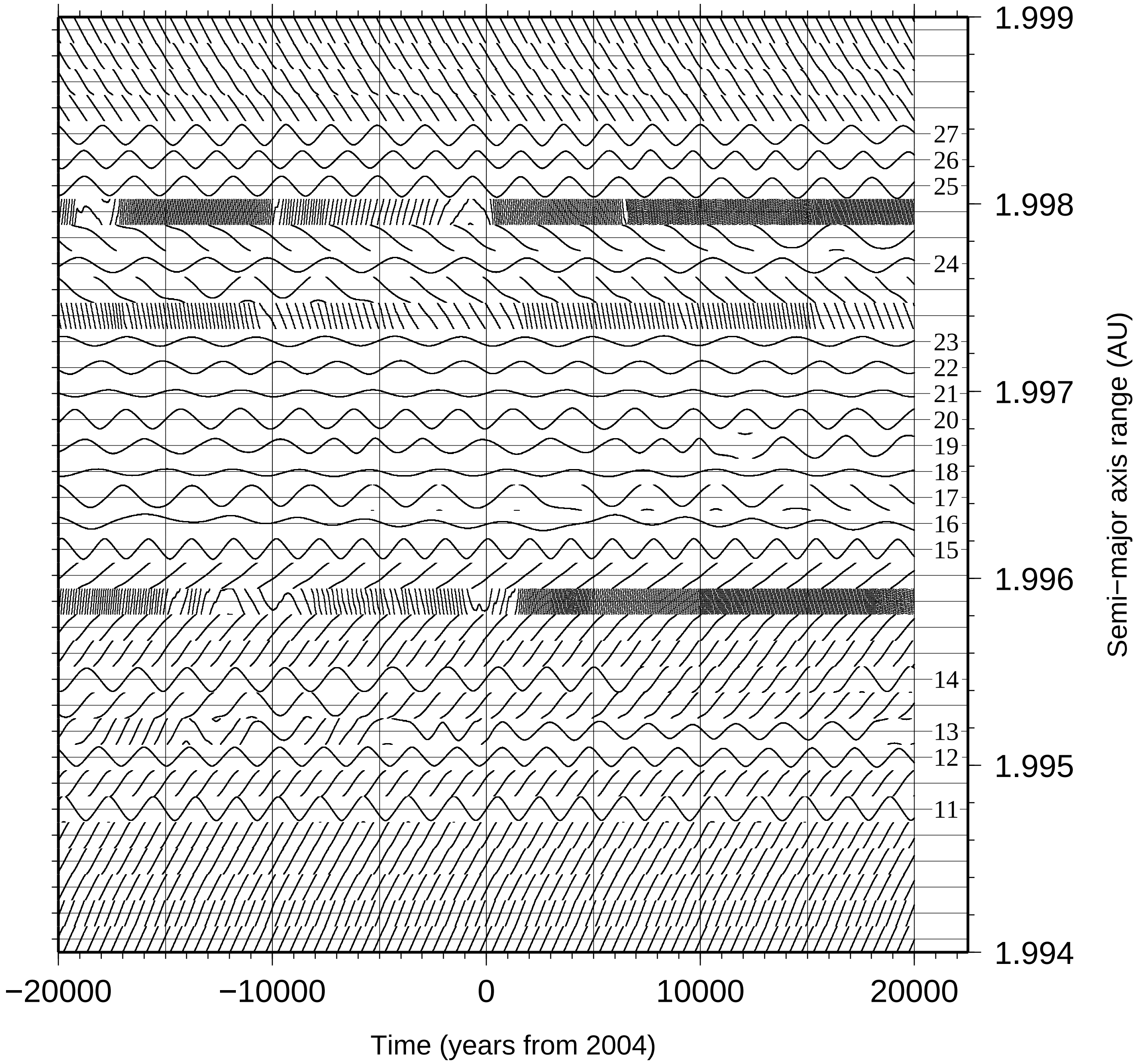
Inclination (degs)
9.03
9.02
9.01
0.216
0.215
0.214
0.213
0.212
Eccentricity
Semi−major axis (AU)
1.313
1.312
1.311
1.310
1.309
1.308
1600
1800
2000
2200
Time (years A.D.)


Distance to Earth (AU)
0.04
0.03
0.02
0.01
0.00
360
180
0
Resonant argument (degs)
Semi−major axis (AU)
0.950
0.948
0.946
0.944
−6000
−5000
−4000
−3000
−2000
Time (years B.C.)



0.05
0.04
0.03
0.02
0.01
0.00
Distance to Mars (AU)
0.6
0.4
0.2
0.0
Distance to Earth (AU)
360
180
0
Resonant argument (degs)
1.315
1.310
1.305
Semi−major axis (AU)
4000
4500
5000
5500
6000
6500
7000
Time (years A.D.)


1.316
1.315
1.314
1.313
1.312
1.311
1.310
1.309
1.308
1.307
1.306
e
10
d
9
8
7
c
6
5
−4000
−2000
0
2000
4000
Time (years from 2004)
Semi−major axis range


27
26
25
24
23
22
21
20
19
18
17
16
15
14
13
12
11
1.999
1.998
1.997
1.996
1.995
1.994
Semi−major axis range (AU)
−20000
−10000
0
10000
20000
Time (years from 2004)