A Survey of Orbits of Co-orbitals of Jupiter

by R. Greg Stacey

Athabasca University and University of Alberta

and

Martin Connors

Athabasca University

Abstract: Jupiter's Trojan asteroids fulfill the prediction of Lagrange that orbits can be stable when a small body orbits in specific locations relative to its 'parent' planet and the Sun. The first such Trojan asteroid was discovered in 1906 and subsequently similar asteroids have been discovered associated with Mars and with Neptune. To date no Trojans have been discovered associated with Earth, but several horseshoe asteroids, co-orbital asteroids moving along a large range of the Earth's orbit, have been found. Other planets also are not known to have Trojan-type asteroids associated with them. Since the number of detected Jupiter Trojans has increased dramatically in the last few years, we have conducted a numerical survey of their orbital motions to see whether any in fact move in horseshoe orbits. We find that none do, although there is some possibility that escaped Trojans have been detected. Here we also use the enlarged database of information about Trojans to summarize their orbital properties as now known.

Résumé: Les astéroïdes Troïens de Jupiter vérifient la prevision de Lagrange que des orbites stables sont possibles lorsqu'un objet de petite masse fait son orbite dans certains endroits par rapport à une planète associée et au soleil. Le premier de ce type d'astéroïde fut découvert en 1906 et ensuite de tels astéroïdes ont été découverts associés à Mars et à Neptune. Jusqu'au present il n'y a pas d'analogue des Troïens associé à la terre, mais plusieurs astéroïdes avec des orbites en 'fer de cheval', qui se déplacent le long de l'orbite de la terre, ont été trouvés. Les autres planètes n'ont pas d'asteroïde suivant leurs orbites. Avec l'accroissement dramatique du chiffre des Troïens de Jupiter connus dans les dernières années, nous avons fait un réperage numérique des mouvements orbitaux afin de determiner si certains se déplacent en fait dans des orbites à fer de cheval. Nous trouvons qu'il n'y en a aucun qui fait ce genre d'orbite en ce moment, mais il existe la possibilité que certains objets soient des Troïens echappés. En plus de presenter nos resultats à ce sujet, nous profitons de notre étude pour presenter les propriétés des orbites des Troïens de Jupiter en general.

1. INTRODUCTION

Since the first Jupiter Trojan was discovered in 1906, many other similar objects, following or preceding Jupiter along its orbit, have been discovered. The three possible classes of co-orbital motion, Trojan libration near a Lagrange point, horseshoe motion along the planet's orbit, and quasisatellite libration in the vicinity of the planet, have now been observed in the Solar System. However, most objects associated with Jupiter appear to be Trojans. The number of co-orbital companions of Jupiter being very large, that they should all be restricted to this class of motion deserves investigation. The increase in number of known Jupiter Trojans has been dramatic in the past decade, as observing and computing power available to astronomers has improved. Despite the dynamics of Jupiter Trojans having been explained analytically for the past two hundred years, there remain many questions about them. The overall question of their origin and relation to other Solar System objects remains tantalizing. For individual objects among the rather large number now known, the long-term dynamics, which can be related to questions of origin and fate, can be hard to determine. To study this requires a large-scale numerical survey of the Jupiter Trojan swarms, and an attempt to classify and group the motions currently undergone by these bodies. We did this with special attention to asteroids potentially undergoing horseshoe motion in Jupiter's co-rotating frame.

2. BACKGROUND

Trojan asteroids, by being at very nearly the same semimajor axis as the associated planet, share its mean motion and thus orbital period. The circumstances of gravitational interactions between such bodies then repeat periodically, and are resonantly amplified. Jupiter, because of its relative size and proximity to the main asteroid belt, is the dominating partner in most asteroid resonances. These resonances can serve to destabilize certain orbits, such as the Kirkwood gaps found in the main belt (Hadjidemetriou, Tsiganis, and Varvoglis, 2002). Conversely, resonances are also capable of producing stable orbits, as is the case with the Trojans, which have a 1:1 resonance with Jupiter. The details of this resonance largely determine the orbital behaviour of any given Trojan asteroid.

To examine the dynamics of asteroid motion analytically, it is useful look at an approximate model which contains only the Sun, a planet, and an asteroid which behaves as a test particle. Usually these bodies are considered to move in a plane and the Sun and planet to move in circular orbits. According to Lagrange's work of 1772, the restricted three-body model predicts five positions relative to the planet, where the test particle can remain in a stable orbit (Murray and Dermott, 1999). These are called Lagrange points, and traditionally denoted by the capital letter L with a subscript. L_1 lies between the planet and the sun, L_2 lies behind the planet on a line connecting it to the Sun, and L_3 lies directly opposite the planet on the other side of the Sun. All five Lagrange points are technically stable, the contours of effective potential near L_1 are saddle shaped, L_2 and L_3 are at peaks, and only L_4 and L_5 are in bowl shaped effective potential wells. Only L_4 and L_5 are stable in the sense that an object moving slightly away from them would not continue to move away. These two points lie 60º away from the planet along the orbit, in the coordinate frame rotating at the rate of the planet's revolution around the Sun. Zero-velocity curves arise in the restricted three-body problem. These curves are related to the effective potential but do not represent the actual orbits of small bodies. Nonetheless, they show the two general classes of motion, which are tadpole orbits and horseshoe orbits, based on the appearance of the associated zero-velocity curves in this diagram. In the case of Jupiter, only Trojan asteroids have been known to date. In the case of Earth, likely due to observational selection effects, only horseshoe objects are known (Brasser et al. 2004). In the case of Mars, it has recently been realized that both Trojan and horseshoe objects exist (Connors et al., 2005).

The known Jupiter Trojans lie in two main swarms along Jupiter's orbital path, as can been seen from Fig. 1. As can be seen in Fig. 1, the Trojans have a noticeable vertical dispersion. The view from the side allows one to see that the orbits must be tilted at angles up to and even beyond about 30º. Despite this tendency to large inclinations, a theory of three body motion still can be applied, with some complications beyond those of the restricted problem but still with similar results.

There may also be a difference in the number of objects librating around each Lagrange point (Fig. 2) – approximately three fifths of the currently accepted Jupiter Trojans are known to be in the L_4 swarm. More specifically, of the 1618 Trojans whose orbits we studied, 1014 were near L_4, and 604 near L_5. As of February 2005[1],1066 L_4 and 660 L_5 Trojans were known, 52 new L_4, and 56 new L_5 objects having been recently discovered despite L_5 being usually higher in the sky. This disagreement was more pronounced prior to recent more numerous observations, and it drew the attention of theorists. In 1989, it was pointed out that bright Trojans were equally numerous in the L4 and L5 swarms - the discrepancy lay in the number of dim Trojans observed, possibly caused by numerous asteroidal collisions in the L_4 swarm (Shoemaker, Shoemaker, Wolfe). As well, Milani (1993) conducted a study on families in the Trojan swarms, groups of objects with similar characteristics which are possible fragments of collisions, and found that the L_5 point was lacking in significant families while the L_4 swarm contained approximately four three-or-more groupings and 3 couples. In contrast, the L_5 swarm contained only one triplet and no significant couples. With recent studies, it is becoming apparent that it is possibly more than just an observational discrepancy (Schwarz, Gyergyovits, Dvorak, 2004).

Motion in the restricted model consists of apparent short-period loops of period close to that of the planet, superimposed on a longer-term drift known as libration (Fig. 4). This is described in standard works such as Murray and Dermott (1999), and the short-term loops can be well modeled as 'epicycles' which would arise to zero order even if there were no gravitational interaction between the planet and the co-orbital body. The libration has a longer period and can be regarded to first order as originating through the motion of the co-orbital body in a potential well (Érdi, 1997).

In order to determine the positions and motions of solar system objects, a numerical integration package is required. For this study, the Mercury integration package of Chambers (1999) was used, with the Sun and all the planets used for the integrations. A hybrid symplectic/Bulirsch-Stoer integrator algorithm was used within the Mercury package, as this algorithm is capable of computing close encounters, necessary to deal with horseshoe objects. In fact, we noted no such close approaches, consistent with our conclusion below that there are no Jupiter horseshoe orbiters currently known.

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[1] 1120 at L_4 and 747 at L_5 as of February 2006

3. STATISTICAL PROPERTIES

Some previous studies attempted to find relationships among parameters of the Trojans (Bien and Schubart, 1987). Such studies often benefit from removal of short-term perturbations which characterize the osculating orbits. Averaging theories generally produce proper elements which are close to invariants of the motion. In the case of Trojans, heavily locked into resonance, there are three proper elements, the proper eccentricity, the proper inclination, and the amplitude of libration (Marzari, #). These have been used among other things to identify possible families, likely of collisional origin, among the Trojans.

In the course of our integrations of many Trojan orbits, and with our focus on finding horseshoe objects, we focused on amplitude of libration. Here we present results that may be compared to the predictions of a theory of the motion of Trojans going beyond the restricted three-body problem (Érdi, 1978).

In Trojan libration around L_4, Érdi (1978) considered the time-averaged longitude difference (in Jupiter's orbital plane) from Jupiter to be α_0 and found an expansion giving this as a function of an expansion parameter l:

$$\begin{aligned}\alpha_0 &= \frac{\pi}{3} + \frac{3\sqrt{3}}{2^3} l^2 + \frac{13\sqrt{3}}{2^8} l^4 + l\cos\phi \\ &- \left(\frac{\sqrt{3}}{2^3} l^2 + \frac{\sqrt{3}}{2^8 3^2} l^4\right)\cos 2\phi + \left(\frac{5}{2^6} l^3 - \frac{65}{2^{12}} l^5\right)\cos 3\phi \qquad (1) \\ &- \frac{25\sqrt{3}}{2^7 3^2} l^4 \cos 4\phi + \frac{1283}{2^{12} 3\cdot 5} l^5 \cos 5\phi + O(l^6)\end{aligned}$$

with $\phi = \sqrt{\tfrac{27}{4}(1 - \tfrac{3}{8} l^2 - \tfrac{97}{512} l^4)}\,u + \delta$, where u is an angular expansion parameter and l a constant of integration. A parameter related to the mass ratio μ often arises in the three-body problem and in this case expansions were done in terms of $\varepsilon = \sqrt{\mu}$, with $u = \varepsilon(\upsilon - \upsilon^0)$, where υ is the true anomaly of Jupiter. To find the limits of the longitude difference, and thus the amplitude of libration, we used values of 0 and π for l. Érdi found the period of libration to be given by

$$T_l = \frac{T_J}{\varepsilon\sqrt{\tfrac{27}{4}(1 - \tfrac{3}{8} l^2 - \tfrac{97}{512} l^4)}} \qquad (2)$$

where T_J is the orbital period of Jupiter. Érdi used T_J = 11.862 years and μ = 0.030885, compatible with modern values. Our computations reproduced values given in his paper to more than the significance given. We then proceeded to compare the values obtained from our orbital integrations to the theory. We made time series of the angular separation of each Trojan studied from Jupiter, and used a simple algorithm to find the extrema of this series. Strictly speaking this corresponds to finding the libration limits for osculating elements. We found the periods of libration by inspection of when the angular distance repeated its extreme values.

Figure 6 shows the amplitudes of libration plotted as a function of libration period. The general trend is that the libration period increases with libration amplitude, a result found for considerably fewer (i.e. 40) objects by Bien and Schubart (1987). We have overplotted the theoretical result described above, based on the theory of Érdi (1978). Since Érdi's theory is for objects coplanar with Jupiter, that is with inclination 0, we have graphed asteroids with inclination greater than 7º separately ###. It may be seen that Érdi's results match very well with the results of our orbital calculations, for those objects with low inclination. It is clear, since most objects of large inclination have longer libration periods than objects of the same amplitude of libration that have low inclination, that libration period is an increasing function of inclination.

The corrected amplitude of libration *d'* is

$$d' = sqrt(C\^{}2 + (0.5\Delta z)\^{}2)/r$$

where C is the fraction of the orbit of Jupiter the object traverses in Jupiter's corotating frame when projected onto Jupiter's orbital plane. The equation $C=rd$ comes from equating the fraction of circumfrence to the fraction of angle transcribed by the object in Jupiter's corotating frame, where r is the radius, and d is the measured amplitude of libration. The extent of motion in and out of Jupiter's plane (i.e. in the z-direction) is given by Δz. This can be used roughly to correct for inclined objects in Erdi (1997), As the motion in and out of Jupiter's plane acts within a short timescale of approximately the orbital period of the object, this motion is averaged away over the longer timescale of the period of libration.

Other recent attempts have been made to explain the absolute magnitude distribution of the Jupiter Trojans (Jewitt et al., 2000; Yoshida, Nakamura, 2005). Assuming a visual albedo of 0.04, the mean visual albedo for the Jupiter Trojans, this magnitude distribution allows for a size distribution as detailed by Jewitt et al. Using the Minor Planet Center catalogue as of 8 May 2006, 1825 potential Jupiter Trojans are in the semi-major axis range between 4.729 AU and 5.656 AU. The absolute magnitude of these 1825 objects is plotted against cumulative logarithmic frequency count in Fig. 2. Using a least-squares fit to the absolute magnitude distribution two slope parameters m were obtained in the ranges 7-9.5 absolute magnitude and 9.5-12.5 absolute magnitude. Assuming that the radii of Jupiter Trojans follow a power-law distribution $n(r) = Ir^{-q}dr$ such that n objects are within the radius range dr, q is related to the calculated slope parameters m by the relation $q = 5m + 1$ (Jewitt et al., 2000). For absolute magnitude < 9.5, q is calculated as 5.315, which agrees within error with Jewitt's (2000) calculation of 5.5 +/- 0.9 for the same range. For absolute magnitude between 9.5 and 12.5, q is calculated as 2.98, which agrees within error with Jewitt's calculation of 3.0 +/- 0.3 for absolute magnitude > 9.5. This work confirms Jewitt's (2000) note of a break in the size distriution for fainter magnitude objects in the Jupiter Trojan clouds.

According to Marzari et al. (#), the different slopes correspond to distinct populations of large (over 30-45 km in radius) and small Trojans (under 30-45 km in radius). The large Trojan population is assumed to represent leftover material from the formation of Jupiter, while the small Trojan population represents collisional fragments. This means that the Jupiter Trojan population is in a state of flux. Subsequent surveys to (Marzari, #) confirm the distinct magnitude and size distributions as calculated by Jewitt et al. (2000), indicating that the Jupiter Trojans would contain 1.28 x 10^6 objects with radii greater than 1 km, more than the main belt.

The tapering off of the absolute magnitudes above 12.5 indicates incompleteness of the Minor Planet Centre catalogue for small Jupiter Trojan objects. Yoshida and Nakamura (2005) predict a second break in the magnitude distribution at absolute magnitudes above approximately 16, using a deep but limited survey of the L4 Trojan cloud. This second break cannot be confirmed nor denied with information from the Minor Planet Center until fainter objects are observed beyond the current catalogue.

4. ABSENCE OF HORSESHOE LIBRATORS

Our survey of the orbits of Jupiter Trojan swarms was performed in the summer of 2003 based on data from the Minor Planet Center. Using the Mercury integrator package we integrated all known and suspected Jupiter Trojans for approximately ten thousand years forward from the then current date, in an effort to catalogue the motions currently exhibited by the objects. Although all objects had their orbits integrated, particular attention was paid to those in the semimajor axis range of 4.73 to 5.094 astronomical units (AU), and to those in the range 5.329 to 5.449 AU. These ranges represent the extremes of the distribution of Trojan semimajor axes and contained 62 and 35 asteroids, respectively. Objects closer to the nominal value of 5.2 AU were felt less likely to be horseshoe librators.

Objects such as 2002 ES_{77}, 2002 EX_{112}, 1996 AV_{10} and 2003 JG_{11}, which had at the time an observational arc on the order of a few days, displayed as horseshoe librators. However, the observational arc for all these objects was determined to be too small to represent a real orbit. Most of these objects were exceedingly faint and at the limits of observation, and are not likely to be picked up again and identified with future observations. The object 2002 GZ_{36} was also investigated in some detail. According to plots of the semi-major axis over time, it is currently librating as a Jupiter Trojan and is expected to lapse into horseshoe motion in approximately ten thousand years, although this result must be treated with caution: the observational arc for the 2002 GZ_{36} data was fourteen days, longer than the other interesting asteroids but still not long enough to trust. Therefore, there are no real observed objects undergoing horseshoe motion with respect to Jupiter. We note that of the objects mentioned, only 2002 EX_{112} and 2003 JG_{11} remained on the MPS Trojan list as of February 2006.

The asteroid 118624 (2000 HR_{24}) appears to be circulating rather than in 1:1 resonance despite being within the limits within which resonant behaviour is expected. This object was one of the only Jupiter Trojans which was exhibiting behaviour which warranted further investigations, as well as having a fairly well-defined orbit. It had a noticeably smaller semi-major axis than most of the other Trojan candidates; as well, its current x-y position shows it to be on the other side of the Sun from Jupiter, placing it outside of the main swarms (Fig. 1), indicating possible horseshoe motion. However, further integrations and a clone study showed it to be simply circulating and not in fact in any kind of resonance with Jupiter. It is possible that this object is in fact a recently-escaped Jupiter Trojan from one of the swarms, however this is not hinted at by the clone study: as can be seen from the graph, it is more likely that 2000 HR_{24} approximately one thousand years ago had a semi major axis of about 4.5 AU, slightly less than its current value (Fig. 3). This can be deduced from the density of the traces near that time. However it is clear that the traces diverge about 600 years back, and that there are regular interactions with Jupiter between then and now. Based on current uncertainties in the orbit and due to these strong interactions, whose details depend critically on distance to Jupiter, it is clear that we cannot trace the orbit with certainty beyond about 600 years into the past. This is a good example of chaos in action and prevents us knowing much about the origins of 2000 HR_{24}.

It was determined that there are no known Jupiter-associated objects with well-defined orbits exhibiting horseshoe behaviour. The majority of the 1,618 objects were shown to be currently librating about either the L_4 or L_5 points. The search for Jupiter horseshoe orbits continues.

5. DISCUSSION

The survey of the Jupiter Trojan swarms' current motions did not present any real horseshoe librators. With the presence of Earth horseshoe objects (Connors et al.) and the relative abundance of Jovian Trojans it might be expected that there would be horseshoe librators among the Jupiter Trojan swarms. Our extensive calculations showed that this is not the case now or in the near future among known objects.

There are a few possible explanations for this. One is that the large number of objects librating around the L_4 and L_5 points may actually inhibit the existence of higher-energy orbits. There may be an increased chance of close interactions and collisions between asteroids in the large swarms, meaning that objects traveling through the swarms at high speed and high energy are much more likely to be ejected than were the swarms emptier, as is the case for the Mars L_4 and L_5 points. That is, those objects boosted into a horseshoe orbit may be quickly kicked out or removed by collisions in the relatively concentrated Jupiter Trojan swarms. Although numerical answers to this possibility have not been looked at in detail, this is not likely to be a statistically satisfactory explanation, as the huge volume that the Jupiter Trojan swarms occupy means that the chance of collision is still low.

As well, it is possible that the proximity of the swarms to Saturn, and the regular nonresonant pulls the Jupiter Trojans receive from it (Marzari, Scholl 2002), serve to destabilize high energy orbits. However, whether Trojans of the inner planets also undergo regular nonresonant interaction, with Jupiter possibly, and whether this is also a destabilizing effect would require further investigation. There are a number of reasons to believe that the lifetime of high-energy horseshoe orbits would be significantly less than tadpole orbits.

Another possibility is that the horseshoe objects have simply not been observed yet. The high energy objects resulting from a collision are likely candidates for horseshoe motion, as well as the most difficult to detect due to their small size. This difficulty in detecting small, high-energy objects would not be as pronounced with Mars and Earth objects. 2002 AA29, the known Earth horseshoe object (Connors et al., 2005) has an absolute magnitude of 24.085 (Milani et al, 2004.) making it unobservable were it to be placed in the Jupiter Trojan swarms. As well as selecting against high energy collisional fragments the trouble with detecting small distant objects also means that there is a very large population of objects which remain undetected. The number of Jupiter Trojans less than one kilometer in diameter is expected to be many times the number of Jupiter Trojans over this size (Jewitt et al., 2000; Yoshida, Nakamura, 2005).

The indication from the clone study of 2000 HR_{24} that this object is most likely not an escaped Jupiter Trojan is reasonable as the Jupiter Trojan swarms are relatively stable over the lifetime of a circulating object like 2000 HR_{24} (approximately a few hundred to a few thousand years). Although_unlikely, it is possible for a fair number of objects in the solar system to be escaped Trojans, as the swarms have been shown to be in a state of flux as a “dynamically unstable structure” (Levison et al. 1997). It has been estimated by Levison et al. (1997) that there are currently more than 200 evaporated Jupiter Trojans with diameters greater than one kilometer traveling the solar system.

We note that very recent work on the possibility that Jupiter and Saturn themselves were once in a 1:2 mean motion resonance (Morbidelli et al., 2005) suggests an extremely dynamic history for the Trojans. In this case, rather than the current value of about 1:2.5, the respective periods of Jupiter and Saturn would have been in resonance, and of course 1:1 resonant objects associated with Jupiter such as the Trojans, would have also been in 1:2 resonance with Saturn. This situation would have arisen rather early in the history of the Solar System: before it there would have already been Trojans left from the formation of the system and these would have been completely dispersed due to the resonance. The present Trojans would have been captured from distant regions after the resonant condition ended. This theory does explain the rather large inclination distribution of the present Trojan clouds, but suggests that comparisons of composition with those of the outer Solar System should be examined, and that Trojans could be expected to have comet-like compositions.

<u>In the search for practical knowledge behind the Jupiter Trojans, many astronomers have attempted to understand what happens to Trojans after leaving libration with Jupiter, especially with the estimation that there are possibly more than 200 evaporated Jupiter Trojans with diameters greater than one kilometer traveling the solar system (Levison, Shoemaker, Shoemaker, 1997). Because of the relatively small semi-major axis these asteroids will spend a significantly longer time near Earth's orbit than other objects,</u>

giving rise to a concern of a possible collision between Earth and one of these bodies. However, as noticeable in Fig. 1, the Jupiter Trojans have an appreciable average inclination, meaning that they would tend to spend more time outside of the Earth's orbital plane. Statistically, the chances of an evaporated Jupiter Trojan asteroid striking Earth are very low, but it is an important dynamic of the solar system nonetheless, and one which should not go unnoticed.

5. SUMMARY

We set out to perform a large scale survey of the approximately 1,600 known and suspected Jupiter Trojans in search of possible horseshoe librators. It was expected that a small number would be undergoing horseshoe motion, due to the relatively large number of horseshoe objects in much smaller Trojan swarms, such as those of Earth and Mars. However, the only orbits catalogued were tadpole orbits, as well as one shown to be simply circulating, 2000 HR_{24}. This lack of horseshoe orbits among the large Jupiter Trojans was unexpected, although the concentration of the Jupiter Trojan swarms, observational power, or proximity to Saturn could explain this. However, this will not be resolved until the Jupiter Trojan swarms have been observed, examined and catalogued to even fainter magnitude limits than is currently the case.

The large number of objects to survey and the difficulties with automating the survey complicated this study, as every candidate must be graphed and viewed - a simple automated print-out was not enough to determine the exact motions that the object is undergoing. To improve results, it is likely that either a less all-inclusive or a more (cleverly) automated attempt would be required.

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Figure Captions.

Fig. 0. Some examples of zero-velocity curves associated with tadpole and horseshoe orbits, as viewed in a reference frame that co-rotates with the planet (the scale has been adjusted so that the planet is one unit from the Sun). The respective names of orbits are obtained from their characteristic shapes. The L_4 and L_5 points are each encircled by 'tadpole' or Trojan-like curves for which a typical zero-velocity curve is shown. Both the L_4 and L_5 points are encompassed by a 'horseshoe' curve. The width of the orbital regions in the radial direction has been greatly exaggerated for clarity. The L_4 point leads Jupiter in its (counterclockwise) orbit around the Sun, while the L_5 point trails it.

Fig. 1. Three views of the distribution of the 1618 objects classified as Jupiter Trojans as of May 2003, depicted in their positions on JD 2451000.5 (July 6, 1998). The top frame shows the view from above the ecliptic plane, with the Sun at centre, and Jupiter indicated. The L_4 Lagrange point is at right, and all objects move in counterclockwise fashion about the Sun. The L_5 Lagrange point is at the left. Asteroid 2000 HR_{24} is labeled far from the Lagrange points, and discussed further in the text. The middle panel shows the view from 30º above the plane of the solar system, looking in over Jupiter. The bottom panel shows the view looking in past Jupiter toward the Sun and illustrates the rather large vertical extent of the Trojan clouds, associated with the generally large inclinations of the Trojan asteroids. Jupiter and the Sun are not to scale relative to each other nor to the scale of the solar system. The Lagrange points are 5.2 AU from the Sun.

Fig. 2. Magnitude distribution of Trojan asteroids gives an indirect look at their size distribution. Histogram of the magnitude of Jupiter Trojans created using accepted objects as of May 2003. The discrepancy in the number of objects in the L4 swarm and the L5 swarm is apparent. As well, the tapering off towards the higher magnitudes (high magnitudes being dimmer than low magnitudes) shows the observational bias towards brighter objects. Note that the L5 swarm tapers off earlier than the L4 swarm, implying that the discrepancy in swarm size is more pronounced with smaller, hard to detect objects.

Fig. 3. 2000 HR24 is known to be circulating with respect to Jupiter at the present date. In an attempt to establish whether this object is an escaped Jupiter Trojan or not fifty

clones were integrated backwards from the present date for 1000 years. A decisive conclusion cannot be reached as to its origins (due to what is likely a close approach with Jupiter at ~1500 A.D.) although the density of the traces suggest that as of ~1000 A.D. 2000 HR24 had a semi-major axis between 4.1 AU and 4.9 AU. This places it outside of the Jupiter Trojan swarms. Due to the chaos demonstrated here, integrations going further back in time are not likely to be useful.

Fig. 4. Six characteristic tadpole orbits depicted over one approximate libration period (~160 years) in Jupiter's co rotating frame, centred on the sun. Broken into three frames for clarity of the orbits. Jupiter, on the right of each frame, is at approximately 5.2 AU. The asteroids oscillate about the Lagrange points which are located at 5.2 AU and either 60º in front of or behind Jupiter. Note the longer term elongated libration superimposed over the shorter term loops ###. 1996 RX15 is on the upper end of both angular libration amplitude as well as libration period for the Jupiter Trojan population, while 2001 SJ256 is on the lower end of both of these. Jupiter and the Sun are not to scale relative to each other nor to the scale of the solar system.

Fig. 5. A plot of difference in semi-major axis and difference in mean longitude (approximately the difference in angular separation) between Jupiter and seven characteristic tadpole orbits (list objects) over one libration period. As can be seen, four objects correspond to the L4 point (60º in front of Jupiter) and three objects correspond to the L5 point (60º trailing Jupiter).

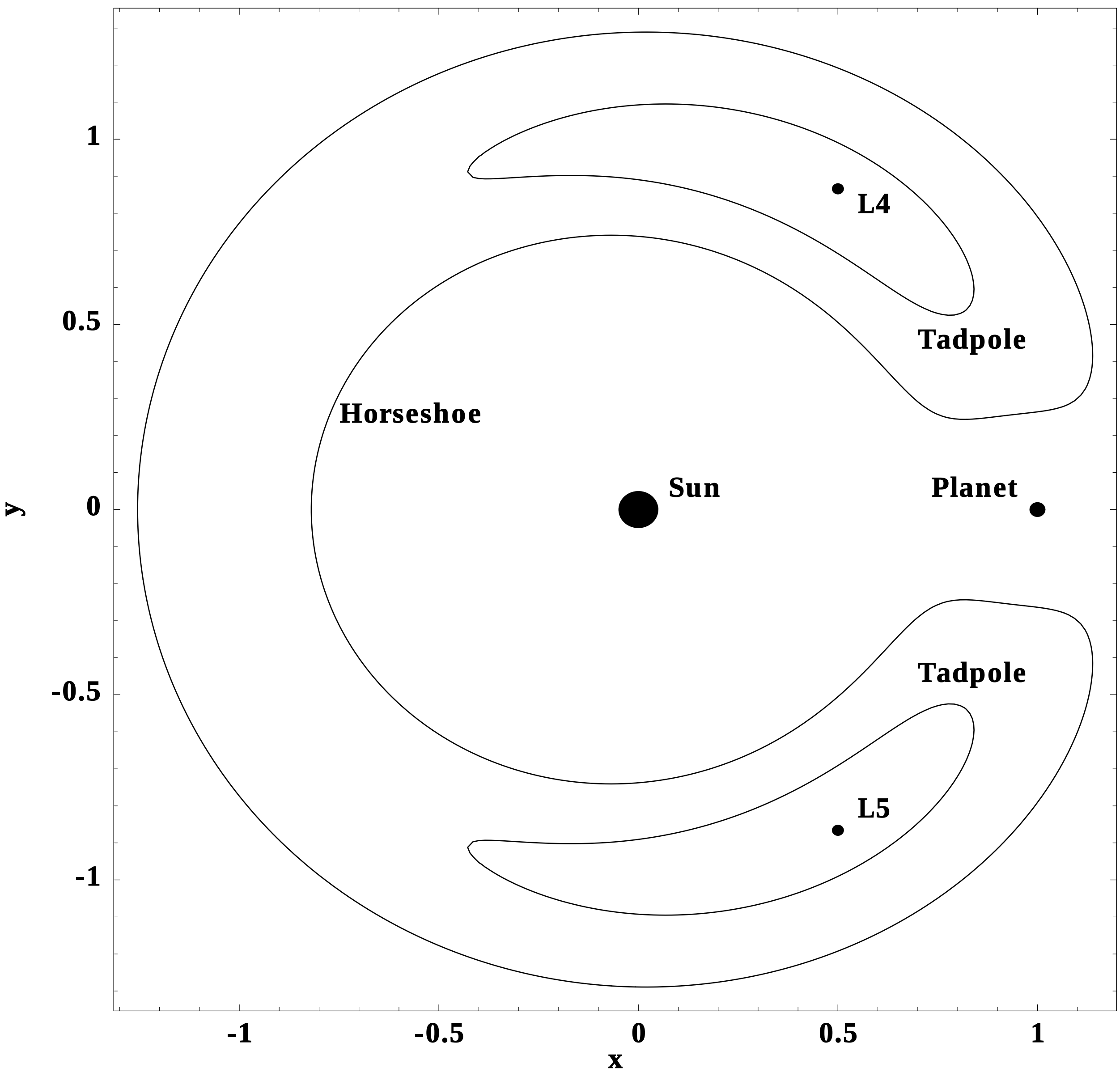
Fig. 6a. A plot of period of libration versus amplitude of libration for 1584 accepted Jupiter Trojans. Libration amplitude and period were calculated over one libration period beginning approximately JD 2451000.5 (July 6, 1998). 34 objects were omitted due to complications in obtaining period of libration and subsequently amplitude of libration. It should be noted that these omitted objects tended to be on the extreme low end of amplitude.
Fig. 6b. The same plot as above, using only the 449 objects with attainable libration period and amplitude which have an inclination under 7º. The theoretical best fit line crosses the period of libration axis at ~147 years, which agrees with theory ###.

To Do

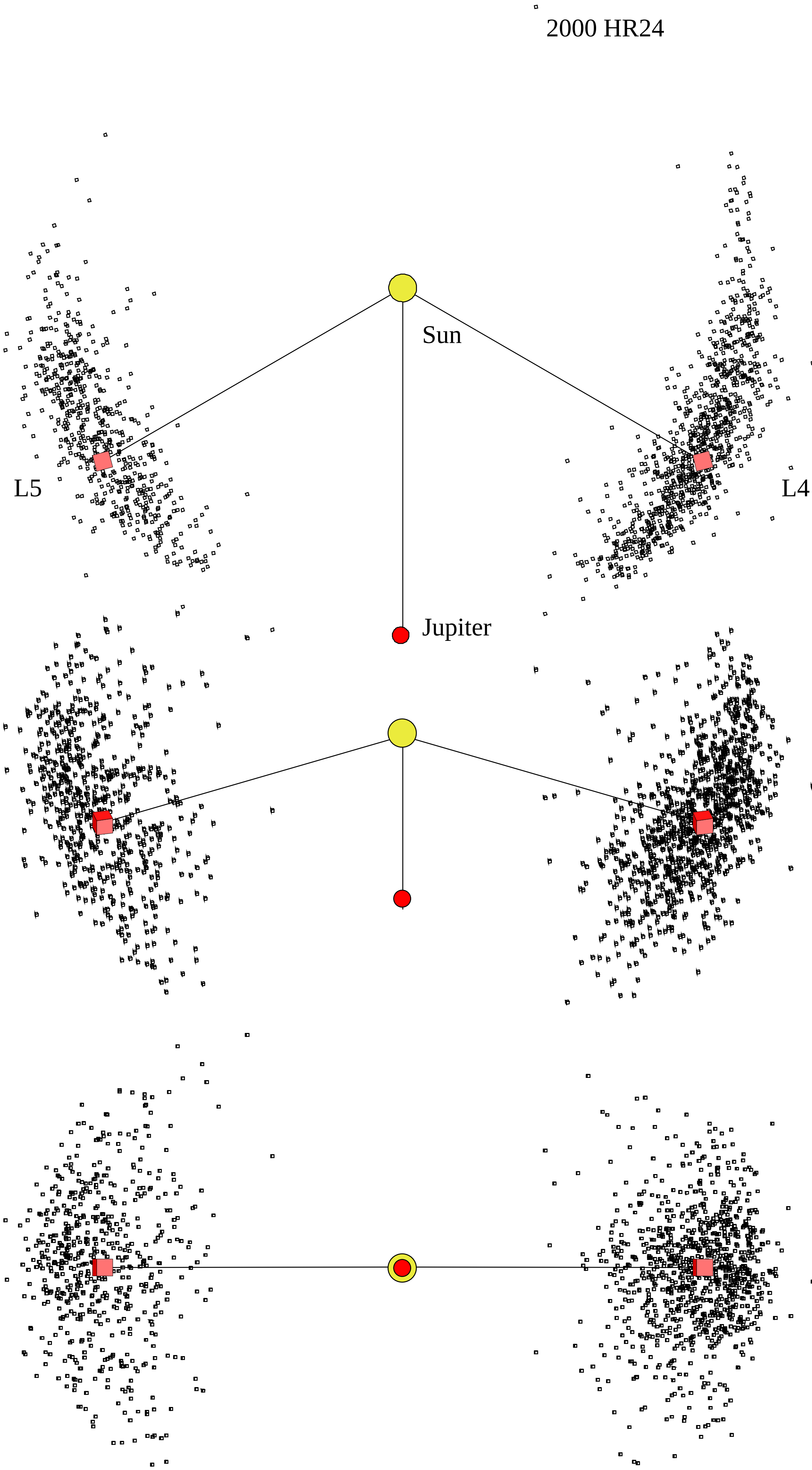
1. Determine exactly how to present figure 6
    a. Two separate figures i.e. high inclination, low inclination? or one figure with separate symbols? or just low inclination?
2. Do we want to include correction of amplitude of libration?
    a. If we do, insert equation
3. Put in correct figure numbering
4.

second to last paragraph in background

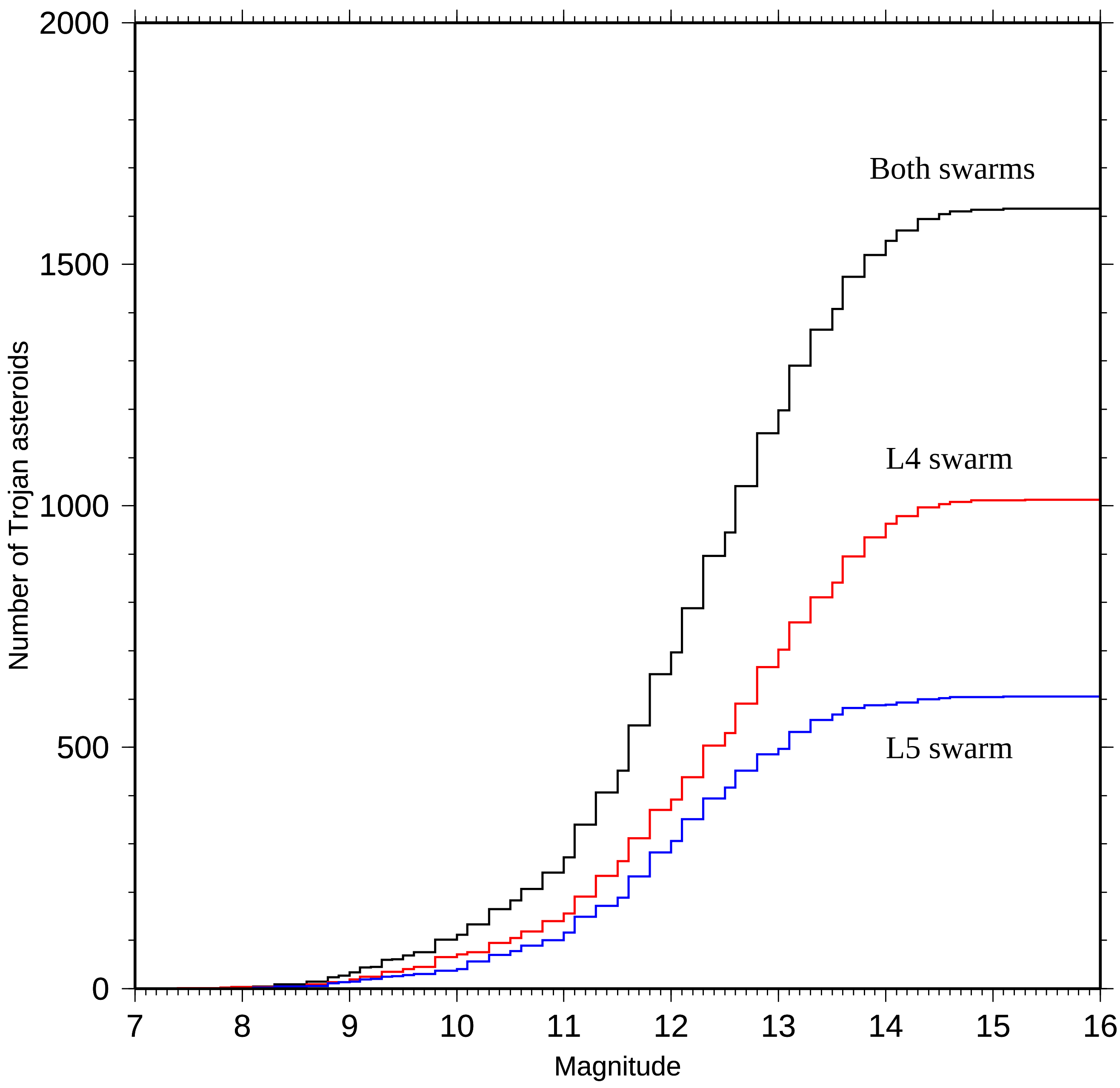
Despite this wealth of analytical theory put forward to explain and describe Jupiter Trojans, there are important questions which are not fully answered. It is thought that there is a relationship between Jupiter Trojan objects and Jupiter Family comets, with the possibility of transition from Trojan to comet and vice versa (Rabe, 1972). As well, the size distribution slope (Fig 2) of the Jupiter Trojans is comparable to that of short-period comets, such as the Jupiter Family comets (Yoshida, Nakamura, 2003). However, despite this speculation the majority of comets, Jupiter Family included, are believed to begin as TNOs possibly in the Kuiper belt (Emel'yanenko, Asher, Bailey; Ipatov, Mather; Malhotra et al.; DUNCAN and LEVISON). Jewitt et al. (200) noted a break in the size distribution for fainter magnitudes among objects which were surely Jupiter Trojans but which had poorly determined orbits.

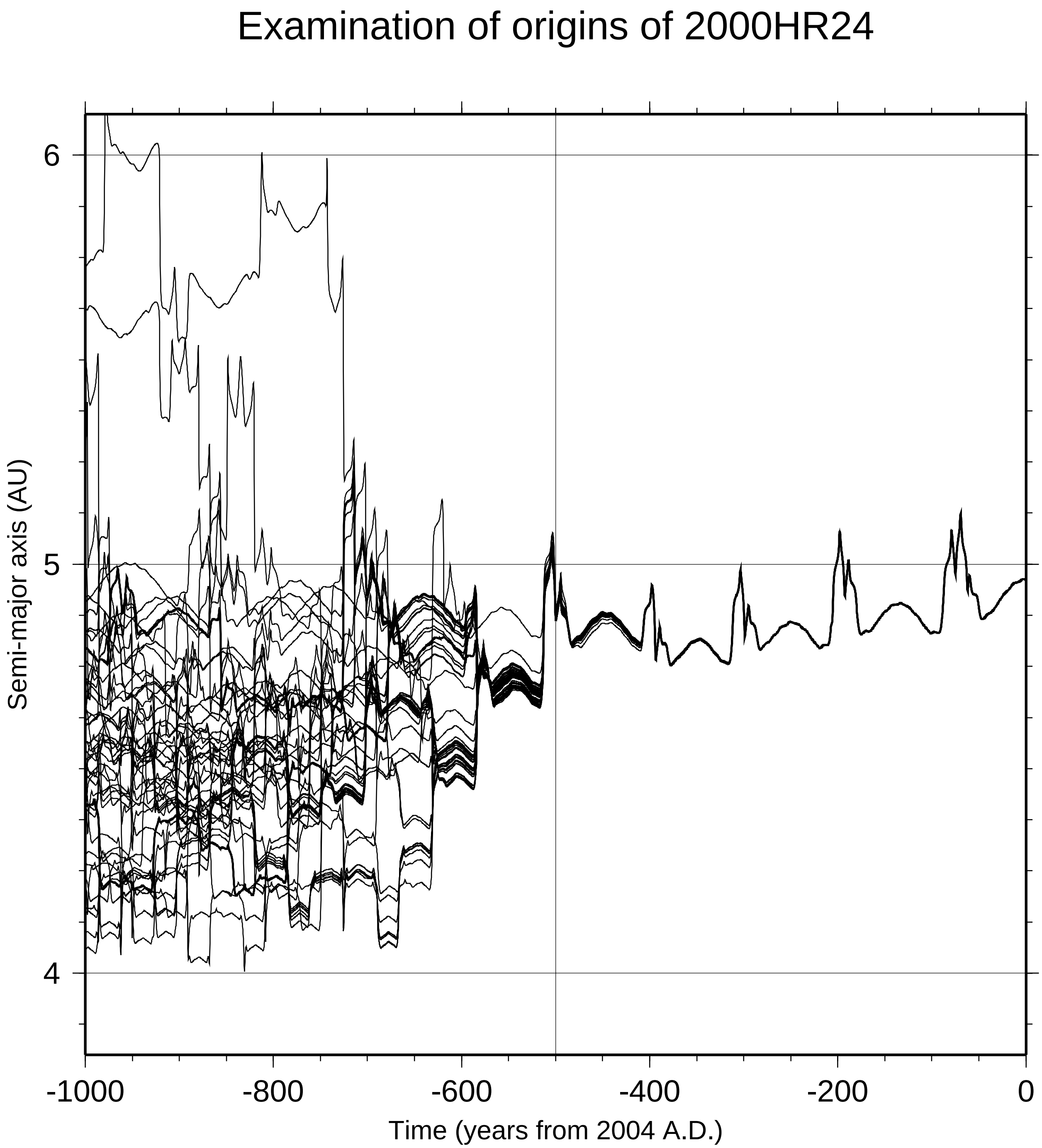


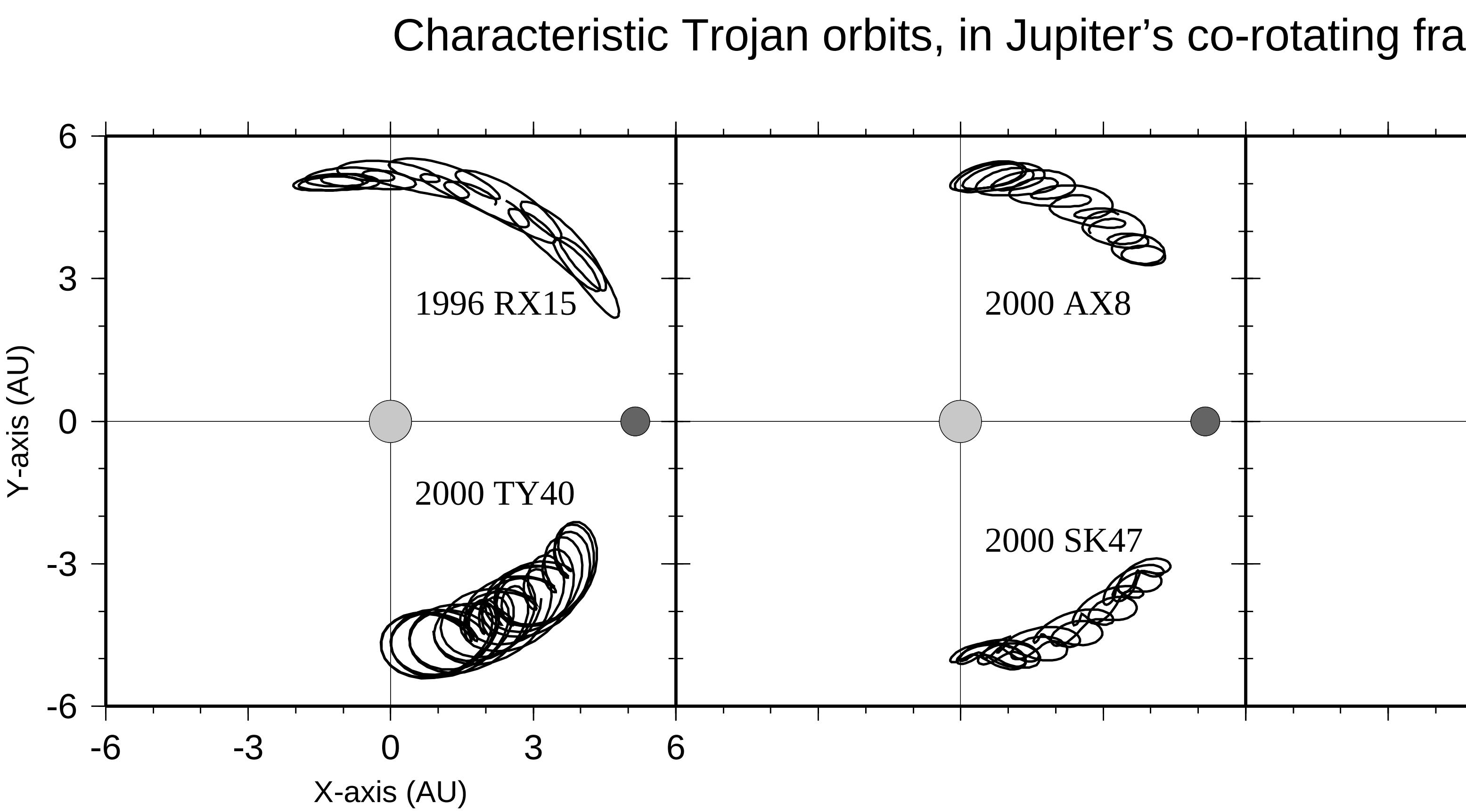
1
0.5
0
-0.5
-1
y
-1
-0.5
0
0.5
1
x
Horseshoe
Tadpole
Tadpole
L4
L5
Sun
Planet

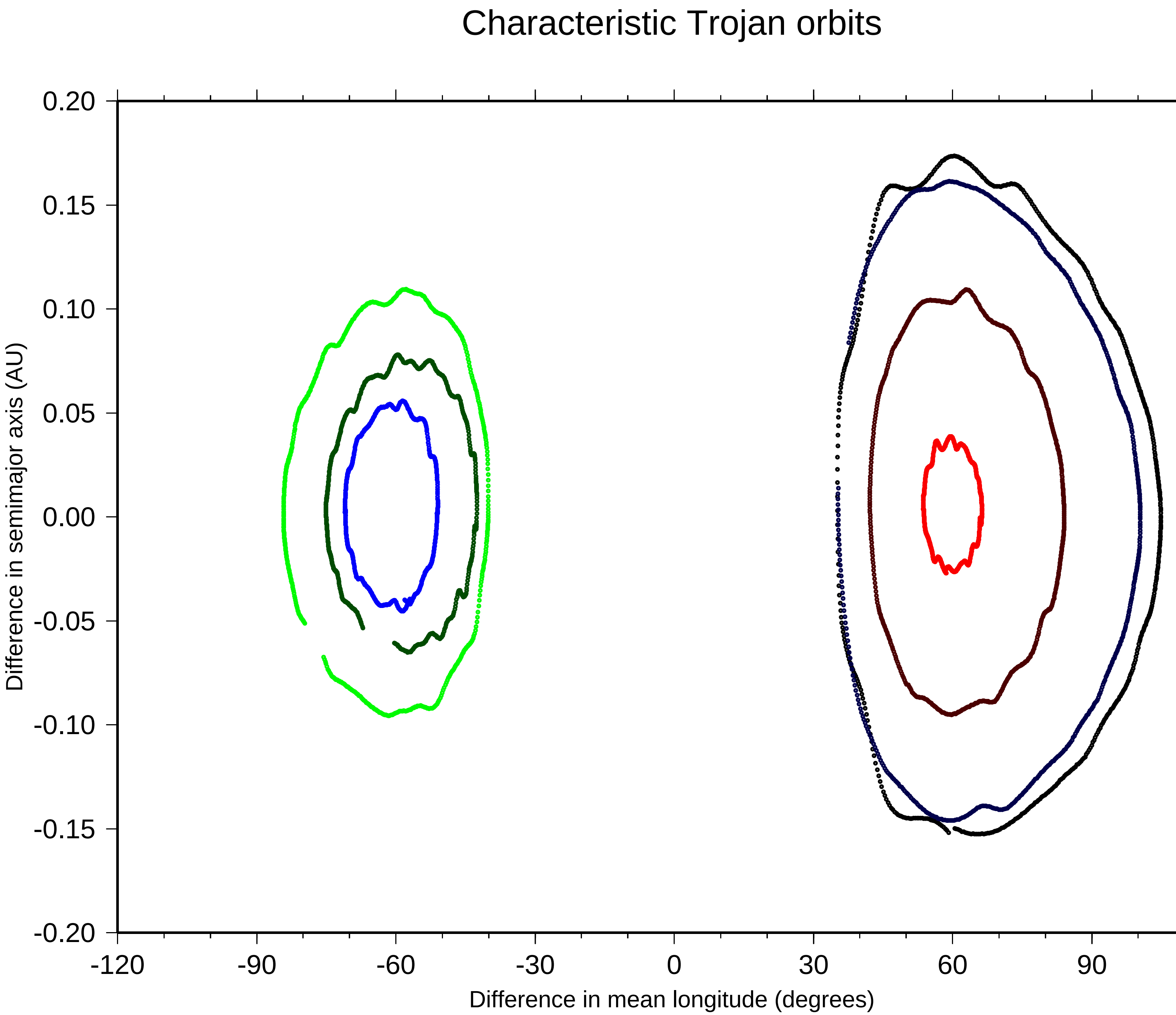

2000 HR24
Sun
L5
L4
Jupiter

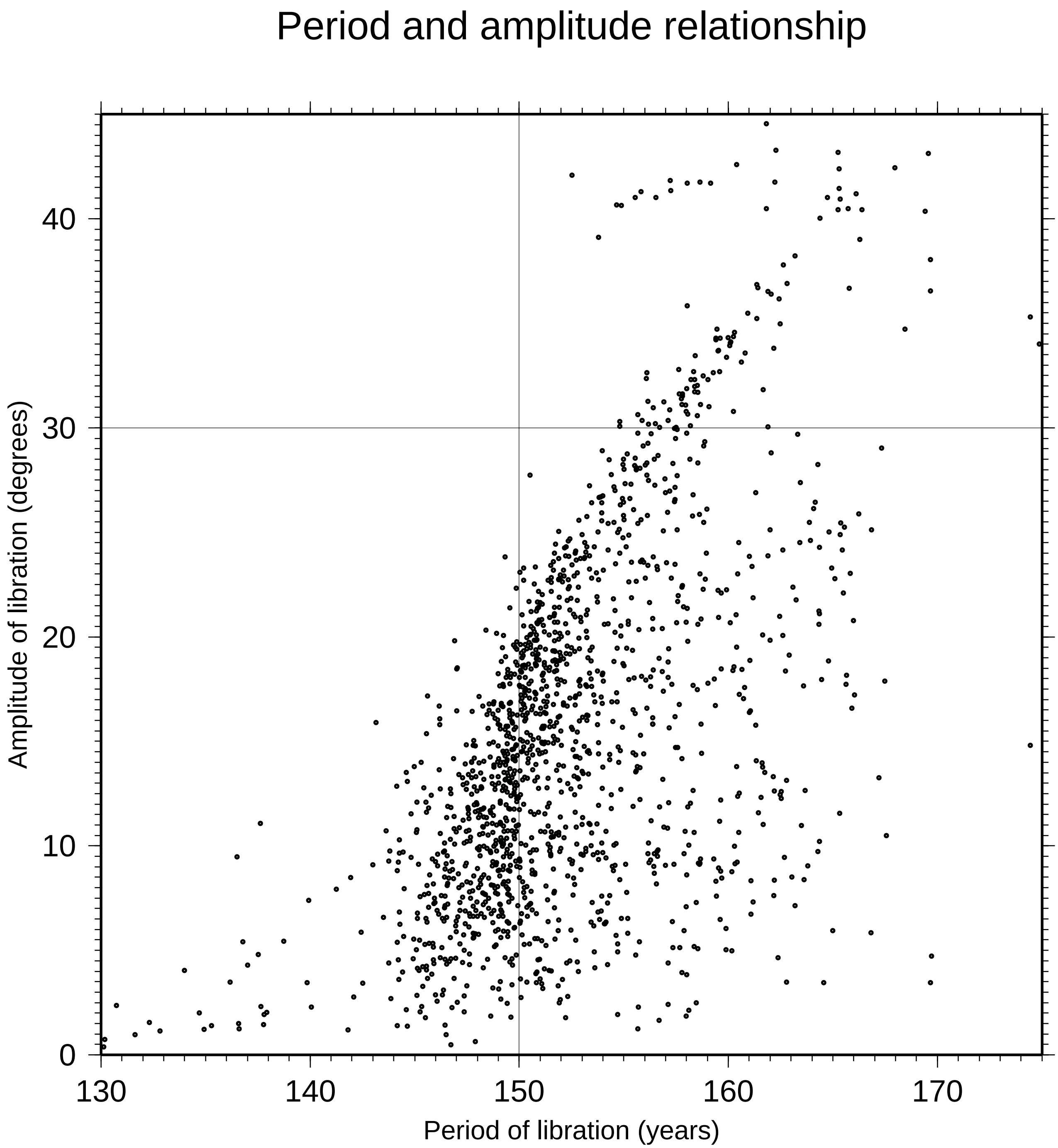
Magnitude distribution of the trojan swarms

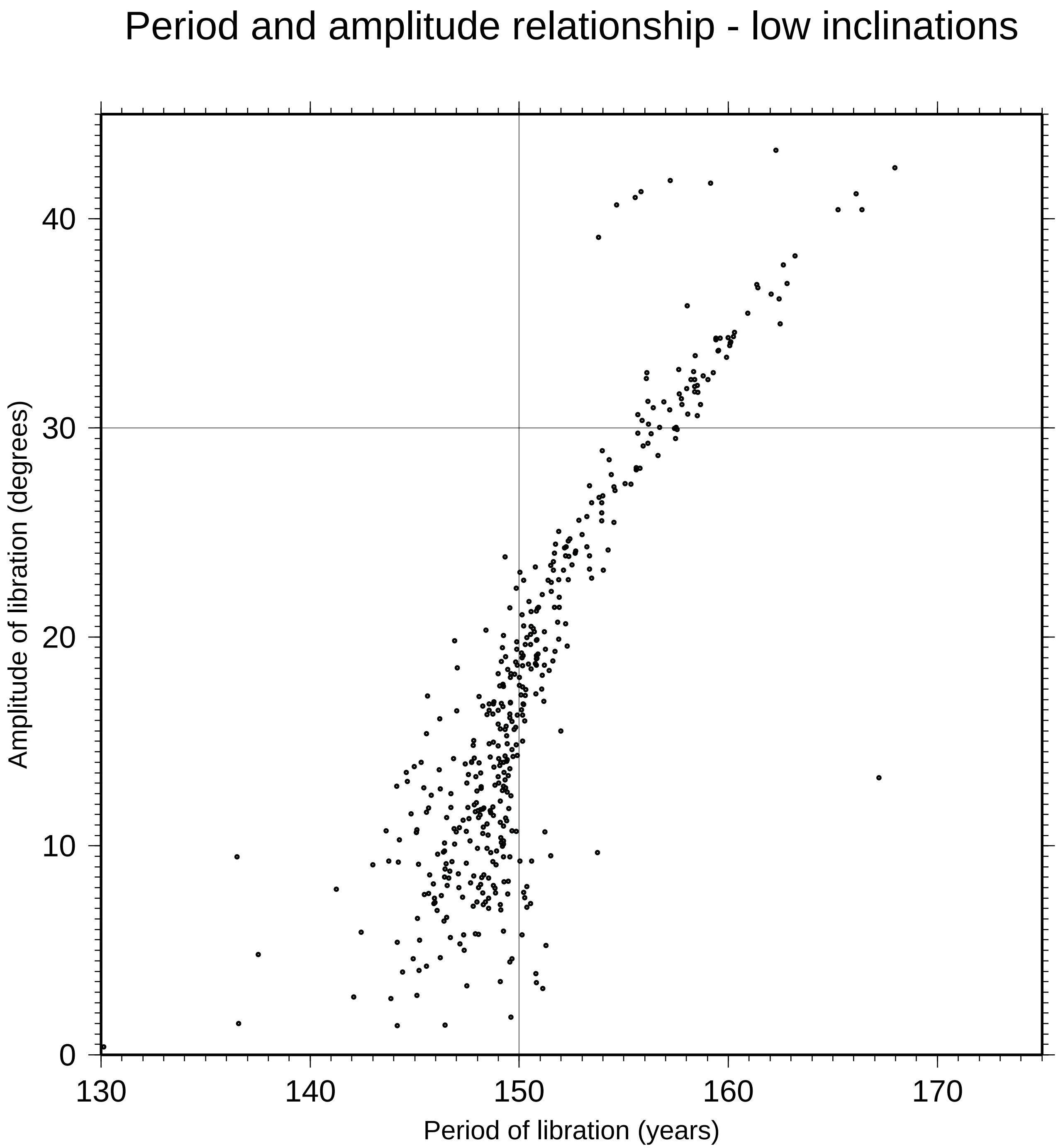



Examination of origins of 2000HR24
6
5
4
Semi-major axis (AU)
-1000
-800
-600
-400
-200
0
Time (years from 2004 A.D.)


Characteristic Trojan orbits, in Jupiter's co-rotating fra
6
3
0
-3
-6
-6
-3
0
3
6
X-axis (AU)
Y-axis (AU)
1996 RX15
2000 TY40
2000 AX8
2000 SK47


Characteristic Trojan orbits
Difference in semimajor axis (AU)
Difference in mean longitude (degrees)
0.20
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
-0.20
-120
-90
-60
-30
0
30
60
90


Period and amplitude relationship
Amplitude of libration (degrees)
Period of libration (years)
0
10
20
30
40
130
140
150
160
170


Period and amplitude relationship - low inclinations
Amplitude of libration (degrees)
0
10
20
30
40
130
140
150
160
170
Period of libration (years)